

## THE TERM STRUCTURE IN THE UNITED STATES, JAPAN, AND WEST GERMANY\*

EDUARD J. BOMHOFF

PETER C. SCHOTMAN

Erasmus University

### I. INTRODUCTION

In this paper, we analyze the term structure of interest rates in the three major capitalist economies. Our principal objectives are:

1. Data reduction - We apply a uniform methodology to analyze the statistical properties of holding-period returns for a variety of maturities on government bonds in the United States, Japan, and West Germany. This will widen the empirical base for research on the term structure beyond the more easily accessible United States data. The data reduction is performed using a latent variable model derived from the Arbitrage Pricing Theory (APT). We will assume that a single latent index can describe the essence of the movements in the yield curve over time.

2. Theory - We use standard asset pricing theories to derive interesting hypotheses about domestic and international influences on the slope of the term structure in each of the three countries.

3. Exploratory testing - We test hypotheses about the changes in the slope of the yield curve. Our dependent variable will be a monthly series for holding-period returns that summarizes the information in the term

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structure.<sup>1</sup> (Data on the term structure for Germany and Japan are shown in Figures 1 and 2.) We regress it on two types of explanatory variables: "news" terms, variables measuring macroeconomic uncertainty, and proxies for risk. A few selected regressions are performed also with "on-line" statistical techniques in order to leave room for different types of uncertainty: agents may have to learn about the appropriate model, the dynamics of the exogenous input variables may change over time, and the character of the output noise may be time dependent.

The paper is organized as follows. Section II describes the construction of the latent variable indices for the three countries. Section III contains a description of the portfolio balance model which will be used to relate the risk premium on long-term bonds to the risk premium on foreign exchange and a number of variance and covariance terms. Section IV contains statistical results for the expected rates of inflation in the three countries. Section V integrates the separate lines of research in a regression model for the term structure index. In that section we also used some multivariate Kalman filter techniques to investigate the importance of time-varying parameters. After a concluding section the paper terminates with a technical appendix which provides information about the data sources.

## II. CONSTRUCTION OF AN INDEX OF THE TERM STRUCTURE

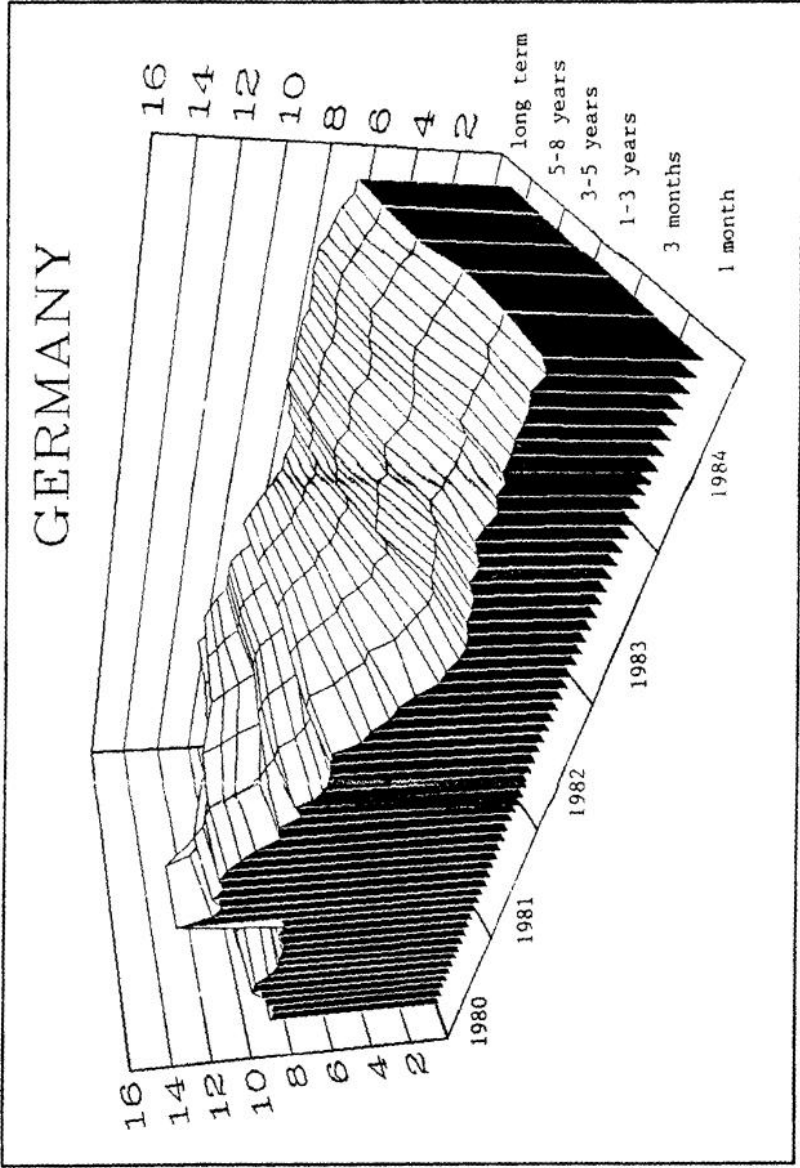
### MOTIVATION

In this section we use the Arbitrage Pricing Theory (APT) to construct a time series that summarizes all relevant information in the term structure. The construction of an index provides the first stage of our empirical analysis. Our motivation for adopting a two-stage methodology is derived from the economic structure of asset-pricing theories.

The starting point for the construction of an index of the term structure of interest rates is the following decomposition of the return on a risky asset:

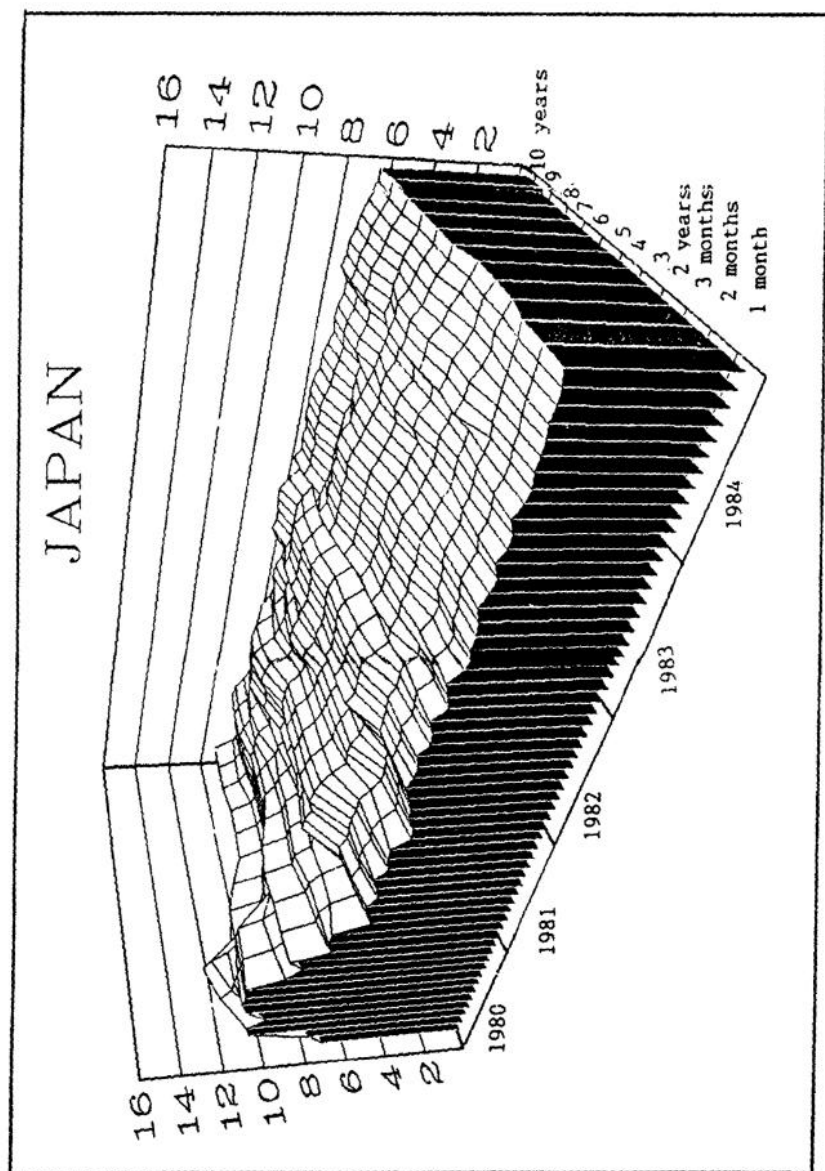
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<sup>1</sup>The alternative approach is to run regressions for yield spreads between long-term yields to maturity and short-term interest rates. Holding-period returns are more appropriate as a dependent variable, since theoretical asset-pricing models are formulated in terms of expected holding-period returns. All the empirical results in the paper could be rewritten in terms of yields to maturity.



YIELDS TO MATURITY

Figure 1



YIELDS TO MATURITY

Figure 2

$$R_i = r_f + r_p + e + u_i \quad (1)$$

where:

- $R_i$  : return on some risky asset  $i$
- $r_f$  : riskfree rate
- $r_p$  : risk premium
- $e$  : non-diversifiable, systematic risk
- $u_i$  : diversifiable, specific risk

The risky part of the return on an asset consists of the random variables  $e$  and  $u_i$ . The only observable part in the decomposition is the riskfree rate  $r_f$ . Asset-pricing theories share the common insight that only systematic risk is priced, implying a link from  $e$  to the risk premium  $r_p$ . By the nondiversifiable nature of this part of the total risk, this link is common for all assets. For the analysis of risk premia the diversifiable component  $u_i$  is irrelevant.

The econometric problem is that the last three components in (1) are all unobservable. Any test of an asset-pricing model will thus be a joint test of the underlying theory and the auxiliary assumptions regarding the statistical distribution of returns, the measurement of some market return, formation of expectations, and the identification of the sources of risk that comprise  $e$ . In most of financial economics risk premia are assumed to be constant, and returns are assumed to be normally distributed. Under these assumptions asset-pricing theories offer testable cross-sectional restrictions for the joint distribution of a set of asset returns; i.e., asset-pricing theory provides equilibrium relations between  $R_i$  and another return  $R_j$ . These simplifying assumptions have been rejected many times, however.<sup>2</sup> The assumption of constant risk premia stems from the fact that asset-pricing theories rarely have any bearing on why risk premia vary over time.

A model of the term structure that reflects this conclusion requires two building blocks. The first is the cross-sectional part, which is within the domain of asset-pricing theory. This part of the model makes

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<sup>2</sup>See Gultekin and Rogalski (1985) for normality in the U.S. term structure, and Bodie, Kane, and McDonald (1983), Mankiw and Summers (1984), and Engle, Lilien and Robins (1987) for examples on time-varying risk premia. Hakkio and Leiderman (1986) find evidence for time-varying risk premia in an international asset-pricing model.

explicit how assets with different risk characteristics have different returns in each period. The second building block is the time series part, which aims at answering the questions we are interested in, like "How do risk premia vary over time?", "What are the relevant macroeconomic news variables?", and "How are the slopes of the term structure related internationally?".

Usually these questions are analyzed by choosing an arbitrary maturity class (e.g., bonds with terms to maturity between 3 and 5 years) and then using a time series of holding-period yields as the dependent variable in regression tests. Engle, Lilien, and Robins (1987), for instance, use the time series of a six-month bond return and a three-month return to provide evidence on time-varying risk premia. These tests can be applied to each maturity class, however. Pooling the information among many maturity classes provides econometrically more informative ways of studying the questions of interest.

Processing such a richer data set, however, will lead to fairly complicated nonlinear multivariate models<sup>3</sup> and therefore causes estimation and testing problems. The cross-sectional relations are well understood and have a strong basis in financial economics. On the contrary, there is as yet only little economic theory on time-varying risk premia. Therefore, misspecification most likely occurs in the time series part of the full term structure model. In analyzing an integrated term structure model, it will be hard to determine statistically which part of the model goes wrong.

This takes us to the arguments for adopting the two-stage approach, suggested by the two building blocks in the theory. First, construction of an index provides a device to split up the cross section and the time series part of the term structure model. The constructed series is the excess return on a (mean-variance efficient) portfolio of bonds with differing terms to maturity. The index series will be used in the second stage of the empirical analysis.

Second, in the construction of the index we extract the essential components of the return on a risky asset: the risk premium ( $rp$ ) and the systematic noise ( $e$ ). The amount of specific noise, unimportant in asset-pricing models, is minimized. Third, the practical advantage of the data reduction stage is that the single equation econometric modeling, which it

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<sup>3</sup>See, e.g., the term-structure model of Bollerslev, Engle, and Wooldridge (1985).

enables, can deal more easily with an uncertain time series specification.

The two-stage approach is fully consistent with the general structure of asset-pricing theories. In the first state we will avoid any statistical assumption that is inconsistent with financial economics or our purposes in the second (time series oriented) stage. There might only be a statistical efficiency loss, because the time series structure has not been imposed in the first stage. This loss must be weighted against the gain in pooling the information from a set of bond returns instead of choosing a single maturity class, and it must also be weighted against the gain of avoiding complicated multivariate techniques that confuse the specific flaws in the model.

Clarida and Campbell (1987) provide an example of the alternative strategy of specifying a complete term structure model. Clarida and Campbell use a latent variable model to test the CAPM. They specify an equation for the latent return on the market portfolio and proceed to derive the cross-sectional parameter restrictions implied by the CAPM. Clarida and Campbell do not pay much attention to the econometric specification of the equation for the market return. Therefore, the most likely interpretation of a rejection of the CAPM parameter restrictions is that their model for the market is misspecified.

## THE STATISTICAL MODEL FOR CONSTRUCTING AN INDEX

We have chosen to use the arbitrage pricing theory (APT) to provide the criteria that the index should satisfy. The APT is a simple but general theory requiring only few assumptions. Further the APT is easy to implement econometrically because of its flexible linear structure. Another argument is that the restrictions of the APT are compatible with other asset-pricing theories like the CAPM and the consumption-based intertemporal CAPM.<sup>4</sup>

The arbitrage pricing theory formalizes the notion that there exists only a limited number of relevant sources of risk that affect the returns on various capital assets. The basic framework of the APT, formulated by Ross (1976), is the cross-sectional linear factor structure:

$$R = E + B e + u \quad (2)$$

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<sup>4</sup>See, for example, Malliaris and Brock (1981), Section 4.11.

where:

- R : (px1) random vector of returns
- E : (px1) vector of equilibrium returns
- B : (pxk) matrix of risk sensitivities,  $k \ll p$
- e : (kx1) random vector of common risk factors  
 $E(e)=0$  ;  $E(ee')=I$ , the (kxk) identity matrix
- u : (px1) random vector of specific risk factors  
 $E(u)=0$  ;  $E(uu')=D$ , a diagonal matrix ;  $E(eu')=0$

The APT provides a link from the common factor risk e to equilibrium returns E. The APT conditions are:

$$E = {}_1\lambda_0 + B\lambda \quad (3)$$

where:

- $\lambda_0$ : short-term riskfree rate, a scalar variable
- $\lambda$  : (kx1) vector of risk prices
- ${}_1$  : (px1) vector of ones

The riskfree rate  $\lambda_0$  is supposed to be the observed short-term interest rate r. In the rest of this section we will assume that  $k=1$ ; a single common factor suffices in pricing all bonds in the term structure. This assumption has some intuitive appeal, since the assets that are studied differ in only one dimension, the time the bonds mature.<sup>5</sup> In discussing the empirical results we will test for the single factor assumption.

The weakness of the APT is that it does not provide an expression of what the risk price  $\lambda$  should look like, nor which economic variables are behind the unknown state variable e. These issues will be taken up in the second stage.

Combining equations (2) and (3) the APT can be rewritten as

$$y = Bz + u \quad (4)$$

$$z = \lambda + e \quad (5)$$

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<sup>5</sup>Gultekin and Rogalski (1985) applied the APT to the term structure in the U.S. and found evidence for more than one factor. But their analysis relies on precisely those assumptions that we challenge: constant risk premia and normally distributed excess returns.



where  $y = R - r_f$  is the  $(px1)$  vector of excess returns. In studying risk premia we concentrate on the single latent index  $z$ , representing the extent to which returns move together.

For  $p$  series of  $T$  observations on excess returns  $y_t$ , we will extract the index time series  $z_t$  of excess returns using the statistical model

$$Y = ZB' + U$$

where:

$Y = (y_1', \dots, y_T')'$ , a  $(Txp)$  matrix of observations

$Z = (z_1', \dots, z_T')'$ , the  $(Tx1)$  index series to be constructed

$U = (u_1', \dots, u_T')'$ , a  $(Txp)$  matrix of normally distributed, and serially uncorrelated errors with a diagonal<sup>6</sup>  $(pxp)$  contemporaneous covariance matrix  $D$

We will avoid making any assumption on the time series behavior of  $Z$ . Instead we assume that  $z_t$ , an element of the latent time series  $Z$ , is a scalar parameter. The time series  $Z$  is treated as a long vector of unknown parameters in order to be free in modeling  $Z$  in the second stage. Further it is assumed that the parameter matrices  $B$  and  $D$  are constant over time. Identification of all parameters entails a normalization of  $B$  or  $Z$  to fix the scale of both.

Having fully specified the stochastic model we are ready to estimate  $Z$ , using maximum likelihood. The likelihood function has the familiar form of a multivariate regression model<sup>7</sup>:

$$\ln L(B, D, Z) = -\frac{T}{2}[\ln |D| - \frac{1}{T} \text{tr}((Y - ZB')'(Y - ZB'))] \quad (6)$$

The unknown parameters are  $B$ ,  $D$ , and the time series  $Z$ . Differentiating with respect to  $Z$  yields the first-order conditions.

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<sup>6</sup>The diagonality of the error covariance matrix is not a consequence of the economic theory but is enforced to identify the other parameters.

<sup>7</sup>Brown and Weinstein (1983) use a similar model. The differences are that they use maximum likelihood conditional on both the matrix  $D$  and the matrix of factor loadings  $B$ , and that they also estimate a time series for the riskfree rate. They use a preliminary factor analysis to estimate both  $B$  and  $D$ . We condition only on  $D$  and use the observed short-term interest rate.

$$Z = YD^{-1}B(B'D^{-1}B)^{-1} = Yw, \quad (7)$$

where  $w = (w_1, \dots, w_p)$  is a vector of portfolio weights. By the normalization degree of freedom in  $B$  we take  $1'w=1$ . Equation (7) shows that, conditional on  $B$  and  $D$ , the index is estimated as a portfolio of the original series. Using the normalization constraint for identification the index is a weighted average with weights determined by the structure of the moment matrix of the excess returns. Each element of  $Z$  is interpreted as the return on a portfolio of bonds having a minimum of diversifiable risk. The latent index  $z_t$  plays the role of the excess return on the market in the CAPM or the benchmark return in the consumption-based intertemporal CAPM.

With  $D$  known estimation of  $B$  and  $Z$  would reduce to a simple principal component analysis of the excess returns  $Y$ . The matrix  $D$  is then used to solve the scale dependence problem that is inherent to the application of principal component analysis. Estimation of  $D$  in the maximum likelihood framework is impossible, since the likelihood function becomes unbounded if one of the elements of  $D$  goes to zero.<sup>8</sup> Therefore, we perform a preliminary factor analysis to obtain an approximate estimate of  $D$ .

The algorithm to construct the index thus contains the following two steps:

- (1) Estimate the standard one-factor model

$$y_t = \bar{y} + Be_t + u_t,$$

where  $E(e_te_t') = I$ , and  $E(u_tu_t') = D$ .

- (2) Use the estimated  $D$  of step (1) to estimate  $Z$  as the first principal component of the scaled data matrix  $YD^{-1/2}$ . The time series  $Z$  and the corresponding factor loadings are the maximum likelihood estimates of (6) conditional on  $D$ .

## EMPIRICAL RESULTS

Tables 1 to 3 provide information about the bond data that are used to

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<sup>8</sup>See Mardia, Kent, and Bibby (1979, page 275).

TABLE 1

## Index Model for the United States

Sample Period 73:1 - 85:12

Maturity	3-6	6-12	1-1½	1½-2½	2½-3½	3½-4½	4½-6½	6½-8½
Category	months	months	year	years	years	years	years	years
mean	0.76	0.88	1.18	0.92	1.08	1.06	0.61	0.48
std. dev.	3.98	7.49	11.5	15.5	21.2	26.1	30.2	37.0
skewness	1.27	0.96	0.74	0.61	0.38	0.45	0.33	0.41
kurtosis	5.47	4.29	5.06	5.15	4.85	3.90	2.93	3.02
NORM(2)	136.6*	143.6*	180.5*	181.8*	156.5*	104.0*	58.7*	63.8*
b <sub>i</sub> (factor model)	0.28	0.59	0.94	1.29	1.73	2.09	2.36	2.77
b <sub>i</sub> (PRINCO)	0.21	0.46	0.74	1.01	1.36	1.64	1.85	2.16
d <sub>i</sub> (s.d u <sub>i</sub> )	2.31	2.42	2.16	0.40	3.94	4.21	10.5	21.9
risk premium	0.20	0.43	0.69	0.94	1.26	1.52	1.72	2.01
portfolio weight	0.006	0.012	0.024	0.937	0.013	0.005	0.003	0.001

\*' means significant at 5% level

Means and standard deviations are reported as percentages per year

All statistics refer to holding period yields

$$\text{skewness: } SK = \frac{1}{T} \sum (y_t - \bar{m})^3 / s^3$$

s = standard deviation

 $\bar{m}$  = mean

$$\text{kurtosis: } KU = \frac{1}{T} \sum (y_t - \bar{m})^4 / s^4 - 3$$

NORM(2): Normality test of Jarque and Bera (1980)

$$\text{NORM}(2) = \left\{ \frac{SK^2}{6} + \frac{KU^2}{24} \right\} \frac{1}{asy} \chi^2(2)$$

PRINCO refers to estimates from the generalized principal component analysis.

Standard errors (d<sub>i</sub>) of the specific noise are computed from the factor model. Risk premia and portfolio weights are computed from the generalized principal component analysis.

TABLE 2

Index Model for West Germany  
Sample Period 73:2 - 86:6

Maturity Category	2 months	3 months	1 - 3 years	3 - 5 years	5 - 8 years	> 8 years
mean	0.23	0.47	1.09	1.86	2.11	1.99
std. dev	0.71	1.34	9.22	14.4	18.8	22.1
skewness	-2.25	-2.40	-0.89	-0.74	-0.89	-0.68
kurtosis	17.8	18.7	2.22	1.31	2.10	1.55
NORM(2)	225.6*	249.6*	54.1*	26.4*	50.9*	28.7*
$b_i$ (factor model)	0.024	0.055	0.701	1.178	1.543	1.774
$b_i$ (PRINCO)	0.022	0.050	0.638	1.071	1.403	1.613
$d_i$ (s.d. $u_i$ )	0.69	1.25	3.78	2.89	3.25	5.81
risk premium	0.03	0.08	1.02	1.71	2.24	2.57
portfolio weight	0.108	0.074	0.104	0.297	0.307	0.111

See Table 1 for explanatory notes.

TABLE 3

Index model for Japan  
Sample period 77:5 - 86:5

Maturity Category	2 months	3 months	4 year	5 years	6 years	7 years	8 years	9 years
mean	0.30	0.44	1.67	1.97	2.30	2.35	2.47	2.54
std. dev.	0.52	0.91	1.29	14.8	16.3	17.4	18.9	20.5
skewness	1.15	0.21	-0.04	0.03	-0.11	-0.39	-0.002	-0.19
kurtosis	7.29	6.81	0.62	1.11	1.91	0.92	0.86	1.15
NORM(2)	265.5*	211.9*	1.77	5.65	16.8*	6.66*	3.35	6.70*
$b_i$ (factor model)	0.013	0.025	1.016	1.158	1.304	1.355	1.470	1.633
$b_i$ (PRINCO)	0.012	0.023	0.961	1.095	1.233	1.282	1.391	1.545
$d_i$ (s.d. $u_i$ )	0.58	0.96	4.06	5.05	4.52	6.15	6.65	6.05
risk premium	0.02	0.04	1.76	2.01	2.26	2.35	2.55	2.83
portfolio weight	0.110	0.078	0.176	0.130	0.182	0.102	0.095	0.127

See Table 1 for explanatory notes

construct the latent indices. The first five lines of the tables describe the properties of the time series for the holding-period returns (net of the short-term interest rate) at each maturity. All data for the returns are averages for a number of bonds in the same maturity class.  $b_i$  refers to the factor loadings, and  $d_i$  is the  $i^{\text{th}}$  element of  $D$ . The risk premium in Tables 1 to 3 is the product of the sample mean of the latent index  $Z$  with the estimated factor loading for each maturity class ( $b_i \bar{Z}$ ). The portfolio weights show the importance of each maturity class in the construction of the latent index.

One of the salient features of the data is the strong rejection of normality, invalidating standard factor analysis. The rejection of normality is due to high fourth-order moments of the data. In our model all nonnormal components are put into the latent index, which can have any distribution and any type of temporal dependence. The measured standard deviations, risk sensitivities (factor loadings), and risk premia increase monotonically with the maturity of the bonds in all three countries. The returns at the shorter end of the maturity spectrum are least correlated with the index.

The portfolio weights are special in the United States where the bonds with remaining terms to maturity of 1.5 to 2.5 years dominate the index. For Japan and Germany the portfolio weights are more evenly distributed over the maturity classes. Since the estimation of the portfolio weights is conditional on the variance matrix  $D$ , some sensitivity tests were performed to see how the index changes when some elements of  $D$  are set at lower or higher values. The portfolio weights of the United States appear sensitive to an increase in the specific variance of the bonds in the 1.5- to 2.5-years maturity class. The resulting series for the index is only marginally affected, though, because of the strong multicollinearity in the excess-return data.

The indices have also been computed for a larger set of assets, including the excess return (in domestic currency) on one-month investments in the foreign short-term asset. The additional two excess-return series did not obtain a significant weight in the computation of the index, because the holding-period returns on foreign investments have a large variance even when compared to domestic long-term bonds. The correlation between domestic and foreign returns is still strong, though.

Figures 3 to 5 show the constructed indices. Table 4 has descriptive statistics for the three latent indices.

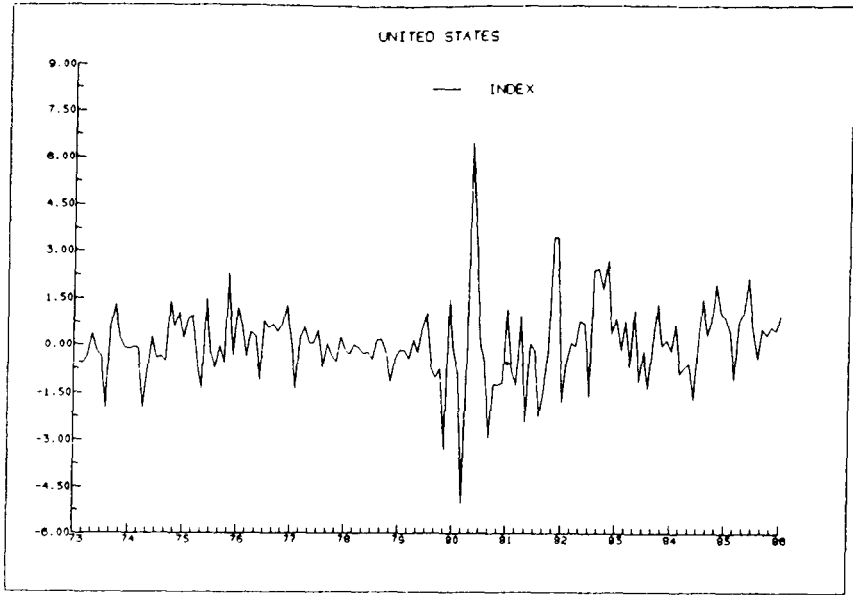


Figure 3

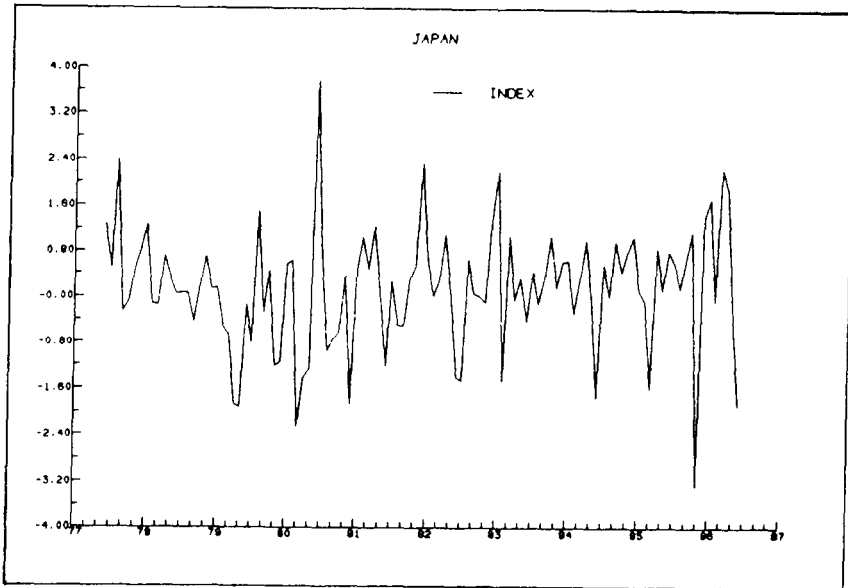


Figure 4

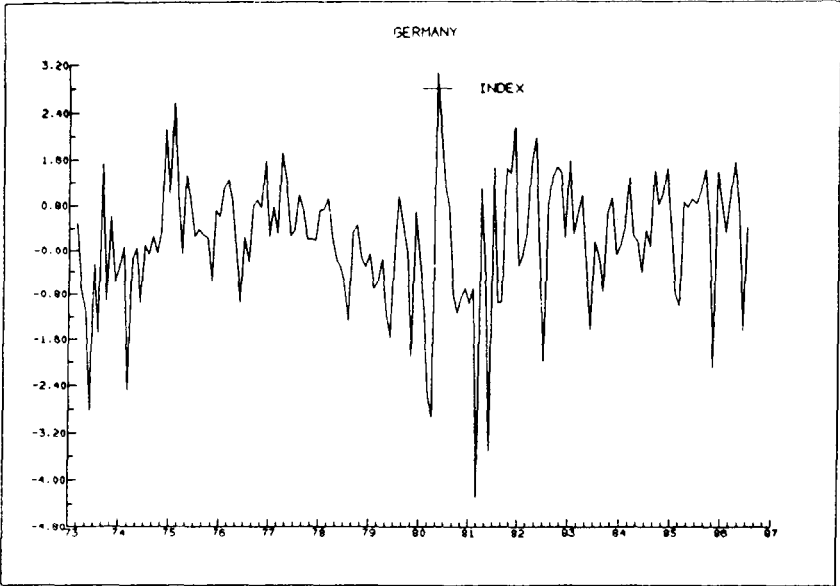


Figure 5

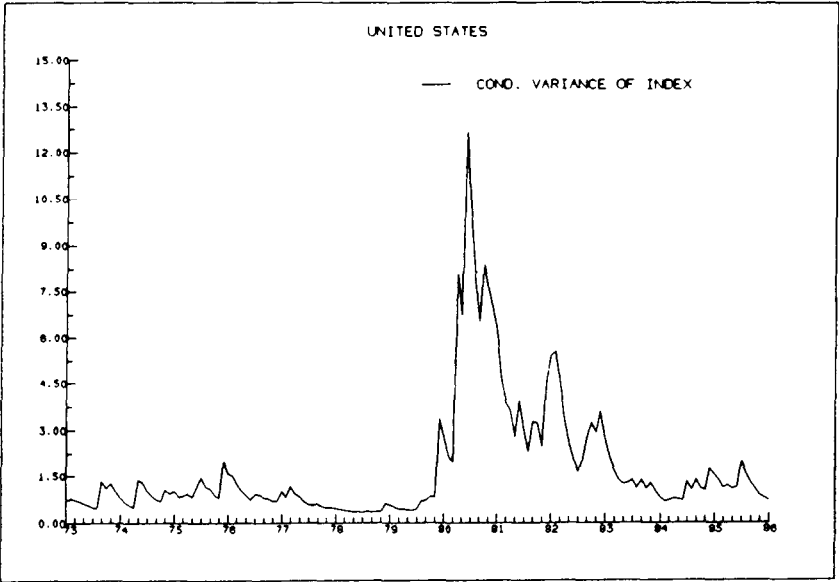


Figure 6

TABLE 4

## Descriptive statistics of term structure index

	United States 73:1 - 85:12	Germany 73:2 - 86:6	Japan 77:5 - 86:5
mean	0.078	0.133	0.153
std. dev	1.28	1.10	1.07
skewness	0.60	-0.84	-0.11
kurtosis	5.02	1.78	1.20
NORM(2)	172.8*	40.3*	6.76*
AUTO(1)	9.04*	9.02*	0.45
AUTO(12)	26.5*	20.1*	12.8*
ARCH(4)	4.55*	3.85*	0.46
ARCH(1)	10.6*	7.9*	1.31

'\*' means significant at 5% level

unit of measurement is percent per month

see Table 1 for definitions of skewness and kurtosis

AUTO(i) : Box-Pierce statistic for  $i^{th}$  order serial correlation ( $\chi^2(i)$ )

ARCH(4) : Modified LM statistic for fourth order ARCH (See Engle (1982)), computed as the F-statistic of auxiliary regression of  $u_t^2 = (y_t - \bar{m})^2$  on a constant and four lags.

ARCH(1) Modified LM statistic for 12<sup>th</sup> order ARCH with linearly declining weights (see Lilien, Engle and Robins (1987)), computed as the F-statistic of auxiliary regression of  $u_t^2$  on a constant and

$$\sum_{i=1}^{12} (12-i)u_{t-i}^2.$$



### III. THE RISK PREMIUM ON LONG-TERM BONDS

#### DESCRIPTION OF THE MODEL

The purpose of this section is to derive a simple expression for the risk premium on bonds in a bilateral international setting. The general model has representative investors in both countries, who hold a portfolio of domestic assets and try to improve the risk return characteristics of that portfolio through international exchanges of assets. We assume that a representative investor in one country (the United States) has the opportunity to trade risk with a representative investor in another country (Japan) in the following ways:

1. The representative U.S. investor can borrow short in the Japanese money market, convert the proceeds to dollars, and invest the dollars short-term in his home country. At the end of a one-month investment period the U.S. investor repays the loan and realizes a pure foreign exchange gain or loss. The two trades in the spot market at the beginning and at the end of the investment period require a Japanese investor who is interested in the opposite set of transactions. Together the representative investors in the two countries determine the quantity of risk traded and the price of pure foreign exchange risk in the foreign currency market. In this way investors in the two countries can trade bets about next period's exchange rate through uncovered positions in the foreign exchange market.<sup>9</sup>

Additionally the U.S. investor may trade in what might be called pure bond risk, i.e., he may borrow short and invest long in U.S. bonds or vice versa (see 2.).

2. The representative Japanese investor may engage in another type of arbitrage: he is allowed to borrow short in the U.S. and to invest the proceeds in U.S. long-term bonds. At the end of the month the foreign investor realizes a gain or loss that depends on excess holding-period returns in the U.S. bond market. Here, we ignore the foreign exchange exposure inherent in this type of trade, since it will amount to a percentage of a percentage. Again, in order to engage in the trade, the

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<sup>9</sup>The model is inspired by Conroy and Rendleman (1983).

Japanese investor requires a U.S. counterpart willing to engage in the opposite trade, i.e., to sell a long-term bond and to invest in the U.S. money market.

The model we will derive is a partial one and considers the amount invested in the market portfolio as given. The return on the market portfolio and total wealth are also assumed to be given.<sup>10</sup>

## DERIVATION OF THE MODEL

We assume that the representative U.S. investor maximizes the expected utility of his wealth at the beginning of period  $t+1$  and that the common mean-variance formulation of the utility of wealth applies. The U.S. investor faces the optimization problem:

$$\begin{aligned} \max_{X,Y} \quad & M^{US} - \frac{X}{W^{US}} rpF - \frac{Y}{W^{US}} rpB - \frac{1}{2} k \left[ \left( \frac{X}{W^{US}} \right)^2 \text{var}F + \left( \frac{Y}{W^{US}} \right)^2 \text{var}B \right. \\ & \left. + 2 \left( \frac{X}{W^{US}} \right) \left( \frac{Y}{W^{US}} \right) \text{cov}(B,F) - 2 \left( \frac{X}{W^{US}} \right) \text{cov}(F, M^{US}) - 2 \left( \frac{Y}{W^{US}} \right) \text{cov}(B, M^{US}) \right] \end{aligned} \quad (8)$$

where:

- $M^{US}$  : expected excess return on U.S. market portfolio
- $W^{US}$  : nominal U.S. wealth
- $rpB$  : risk premium on U.S. bonds
- $rpF$  : risk premium on foreign currency
- $k$  : degree of relative risk aversion
- $\text{cov}(x_1, x_2)$ : covariance between  $x_1$  and  $x_2$
- $\text{var}(x)$  : variance of  $x$
- $X$  : nominal wealth in dollars invested in short-term foreign asset
- $Y$  : nominal wealth in dollars invested in U.S. bonds

Equation (9) formalizes the corresponding optimization problem for the representative Japanese investor.

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<sup>10</sup>Thus the model does not allow for simultaneous determination of consumption plans and portfolio allocation. See Fama (1970) and Long (1974) for seminal discussions of the simplifying assumptions required on the types of uncertainty facing investors if one wants to solve asset-pricing models in a context where consumption and portfolio allocation are determined simultaneously. See Hansen and Singleton (1983) for an empirical study with severe restrictions on both the information set and the types of dynamic uncertainty in the economy. Chan and Stulz (1986) offer a recent comparison between consumption based asset-pricing models and mean variance models.

$$\begin{aligned} \max M^J + \frac{X + CCAB}{W^J} rpF + \left(\frac{Y}{W^J}\right) rpB - \frac{1}{2}k \left[ \left(\frac{X + CCAB}{W^J}\right)^2 \text{var}F + \left(\frac{Y}{W^J}\right)^2 \text{var}B \right. \\ \left. + 2\left(\frac{X + CCAB}{W^J}\right)\left(\frac{Y}{W^J}\right)\text{cov}(B,F) + 2\left(\frac{X + CCAB}{W^J}\right)\text{cov}(M^J,F) + 2\left(\frac{Y}{W^J}\right)\text{cov}(M^J,B) \right] \quad (9) \end{aligned}$$

where:

$W^J$  : nominal Japanese wealth U.S. dollars

$M^J$  : excess return on Japanese market portfolio

CCAB : Japanese accumulated current account surplus in U.S. dollars.

We assume that Japan has accumulated a net foreign asset position which at time period  $t$  amounts to CCAB dollars for the representative Japanese investor. His voluntary additional exposure in the forward currency market amounts to  $eX$  yen ( $e$  is the nominal exchange rate expressed as yen per dollar), so that his total dollar exposure is equal to the dollar value of  $eX + eCCAB$  yen.

In equations (8) and (9) the investors maximize the expected utility of nominal wealth, since we assume that short-term uncertainty regarding the price level in the next period may be neglected. This does not imply that investors are not faced with inflation uncertainty. Rates of return may vary due to changing expectations of future inflation, and covariances between asset returns may deviate systematically from zero because of the effects of changes in inflationary expectations on the domestic financial markets and on the exchange rate. Expected returns on financial assets and the risk premia contained therein depend on the level and variance of the expected rate of inflation, but it is not necessarily the case that one-month-ahead-forecasts of the price level are the one and only relevant variable here. In fact, we shall work with 18-month-ahead forecasts of inflation (see Section II.)

Differentiating equations (8) and (9) with respect to  $X$  and  $Y$  yields the first-order conditions to determine the equilibrium values of  $X$ ,  $Y$ ,  $rpB$  and  $rpF$ . The first-order conditions read:

$$-\frac{1}{W^{US}} rpF - k \frac{\text{var}F}{(W^{US})^2} X - k \frac{\text{cov}(B,F)}{(W^{US})^2} Y = -k \frac{\text{cov}(F, M^{US})}{W^{US}} \quad (10)$$

$$-\frac{1}{W^{US}}rpB - k\frac{\text{var}B}{(W^{US})^2}Y - k\frac{\text{cov}(B,F)}{(W^{US})^2}X = -k\frac{\text{cov}(B,M^{US})}{W^{US}} \quad (11)$$

$$\frac{1}{W^J}rpF - k\frac{\text{var}F}{(W^J)^2}(X + CCAB) - k\frac{\text{cov}(B,F)}{(W^J)^2}Y = k\frac{\text{cov}(F,M^J)}{W^J} \quad (12)$$

$$\frac{1}{W^J}rpB - k\frac{\text{var}B}{(W^J)^2}Y - k\frac{\text{cov}(B,F)}{(W^J)^2}(X + CCAB) = k\frac{\text{cov}(B,M^J)}{W^J} \quad (13)$$

Multiplying equation (10) with  $(W^{US})^2$  and equation (12) with  $(W^J)^2$  and subtracting the two, one obtains the solution for the risk premium in the forward exchange market:

$$rpF = k\frac{W^{US}}{W^*}\text{cov}(F,M^{US}) + k\frac{W^J}{W^*}\text{cov}(F,M^J) + k\frac{CCAB}{W^*}\text{var}F \quad (14)$$

where  $W^* = W^{US} + W^J$ . A similar operation on equations (11) and (13) gives the risk premium for U.S. bonds:

$$rpB = k\frac{W^{US}}{W^*}\text{cov}(B,M^{US}) + k\frac{W^J}{W^*}\text{cov}(B,M^J) + k\frac{CCAB}{W^*}\text{cov}(B,F) \quad (15)$$

An interesting special case arises if the two countries are of equal weight and there are no claims on the U.S. corresponding to cumulated current-account surpluses in the foreign country:  $W^{US} = W^J$  and  $CCAB = 0$ . In this case the risk premium on U.S. long-term bonds is simplified to

$$rpB = \frac{1}{2}k(\text{cov}(B,M^{US}) + \text{cov}(B,M^J)) \quad (16)$$

Under some restrictive conditions the risk premium on U.S. bonds is equal to the simple well-known CAPM expression

$$rpB = k\text{cov}(B,M^{US}) \quad (17)$$

Equation (17) will hold if there is a zero covariance between the return on the Japanese market and the U.S. long-term bond market. Equation (17) will also hold if the foreign country does not possess a cumulated current-account surplus on the U.S. and if the covariance between the Japanese market and the long-term U.S. bond market equals zero. Note that in both these special cases Japanese investors continue to assume positions in long-term U.S. bonds:  $Y$  does not become identically zero. However, the

appropriate slope of the U.S. yield curve is not determined by any other factor apart from the covariance between long-term U.S. bonds and the U.S. market.

#### IV. ADAPTIVE FORECASTS OF MONEY GROWTH AND INFLATION

##### INTRODUCTION

Nominal returns are likely to depend on news about the money supply and the price level. A standard procedure has been to fit univariate ARIMA models to the logarithms of the money supply and a national price index. The residuals of such a model serve as proxies for the unexpected changes in the level. The coefficients in the univariate model indicate which distributed lag on past rates of growth can be used to compute the expected growth rate of the series.

There are two potential problems with this approach. First, why should the time series model be constant throughout the sample period? Second, is the technique robust to outliers in the data?

Below, we shall describe an adaptive method for computing univariate forecasts of inflation. To achieve consistency, the same method is also applied to the time series for the money stock in the three countries.

An essential first question is the time horizon to which the forecast applies: do we wish to use one-month-ahead forecasts of inflation, or should we use proxies for longer-term forecasts? If longer-term forecasts are used, the corresponding forecast errors will exhibit serial correlation at all lags up to the forecast horizon. Unbiased short-term forecasts, by contrast, should result in serially uncorrelated forecast errors. In this research we need proxies for unanticipated short-term movements in the money supply and the price level and the longer-term rates of growth of money and prices. But, how long is the long run within the context of exploring the term structure of interest rates. We conjecture that the risk premia in bond returns may be connected to either the phase of the business cycle or to medium term swings in real money balances (or both). Therefore, we decided to fix the forecast horizon at 18 months for both money and prices. We computed an estimate in each month for the average

rate of inflation (and money growth) for the next 18 months.<sup>11</sup>

## KALMAN FILTER FORECASTS OF LONG-TERM INFLATION

The computational method is strictly recursive ("on-line"). Neither the forecasts nor the coefficients in the forecast model are based on price-level data for periods beyond the present. We use two statistical techniques that satisfy these criteria: the so-called Multi State Kalman Filter (MSKF) and discounted least squares.<sup>12</sup>

The univariate Multi State Kalman Filter has been used before in Bomhoff (1982, 1983) and Bomhoff and Korteweg (1983). Kool (1982) contains an extensive description of the technique. The univariate data are processed in parallel by four or six fixed Kalman filters, each corresponding to an ARIMA (0,2,2) model. The MSKF method generates forecasts which are a weighted average of the separate forecasts, with weights that vary over time according to the prior probabilities of each separate filter. These priors are updated continuously in a Bayesian manner.

The aim here is to generate univariate forecasts of the average rate of inflation over the next 18 months. We proceed in three steps. In the first step the Multi State Kalman Filter imposes an ARIMA(0,2,2) model on the log of the price level, using flexible model coefficients. This procedure results in a time series for the "underlying" price level in each period (the price level minus the estimated purely temporary noise). Let  $p_t$  be the actual price level, and  $Ep_t$  the permanent price level in period  $t$  after the observation for period  $t$  has been processed by the Kalman filter. We form the time series for  $p_t - Ep_{t-18}$  and regress it on a constant and the permanent growth rate as computed in periods  $t-18$ ,  $t-19$  and  $t-20$ . This regression is performed with discounted recursive least squares<sup>13</sup> (discount parameter 0.98; number of periods used for initialization: 10). If the sum of the regression coefficients on the lagged rates of growth is smaller than 1 over the largest part of the sample, this indicates that the

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<sup>11</sup>No experiments were made with any other forecast horizon.

<sup>12</sup>A good general reference to recursive estimation is Goodwin and Payne (1983). Chapter 7 has a very useful overview of recursive least squares and its relationship to Kalman filter methods. See also Ljung and Söderström (1983).

<sup>13</sup>See also the last part of Section V.

rate of inflation exhibits a return to normalcy. We find that the algorithm requires about two years to stabilize; subsequently the coefficients and the constant term behave smoothly. In all three countries the sum of the regression coefficients on lagged rates of price growth converges to a value close to 0.5, suggesting that short-term surges or slowdowns in the inflation rate are not fully permanent but contain temporary elements.

Finally, we use the constant term and the regression coefficients as estimated on the basis of data up to and including period  $t$  to make a forecast of the expected rate of inflation over the period between  $t$  and  $t+18$ .

## V. TIME-VARYING RISK PREMIA AND THE ROLE OF NEWS

### INTRODUCTION

The regressions in this section bring together the separate building blocks developed in Sections II to IV. The dependent term structure variable has been constructed in Section II, a theoretical model risk premia in an international setting has been discussed in Section III, and some important news variables have been constructed in Section IV.

We have divided the time series analysis of the term structure model into four parts. We start with a univariate time series model for the index that describes the variation over time in the risk premium. The second empirical model introduces the short-term interest rate as the most important news variable. Next we explore the empirical importance of other news variables and various proxies for risk premia that are implied by the theoretical model in Section III. Finally, recursive parameter estimates provide an indication that the influence of some of the macro economic factors is not constant over the entire sample period.

### TIME-VARYING RISK PREMIA: THE ARCH-M MODEL

The index constructed in Section II represents the sum of a risk premium and nondiversifiable risk. In the APT the relation between riskiness and risk premium is not made explicit. In the simple one-period mean-variance CAPM, the expected rate of return on the market is related to the variance of the unexpected changes in the market return ( $M$ ):

$$rp(M) = \delta var(M) \quad (18)$$

In terms of the latent index this relation becomes

$$z_t = \lambda_t + e_t \quad (19)$$

$$\lambda_t = \delta h_t + c \quad (20)$$

where  $\lambda_t$  is the price of nondiversifiable factor risk, ( $b_i \lambda_t$  is the risk premium of asset  $i$  in period  $t$ ),  $\delta$  is the degree of relative risk aversion, and  $h_t$  is the conditional variance of  $e_t$ . In a univariate time series model the heteroskedasticity of  $\{e_t\}$  serves as an explanatory variable for the risk premium.<sup>14</sup> To model the conditional variance we adopted the ARCH specification, which is also used in the term-structure model of Engle, Lilien and Robins (1987). Our ARCH specification reads:

$$h_t = E_{t-1}(e_t^2) = \alpha_0 + \alpha_1 h_{t-1} + \alpha_2 e_{t-1}^2 \quad (21)$$

The ARCH model is a mechanical time series approach to model the heteroskedasticity of  $\{e_t\}$ . One of the characteristics of the ARCH model is that the conditional distribution of  $\{e_t\}$  is normal, but the unconditional distribution has fatter tails and higher fourth-order moments than the normal. If the ARCH effect is strong, the fourth moment does not even exist. A time-varying risk premium gives rise to autocorrelation in the index, since the dynamics of  $h_t$  carry over to  $\lambda_t$ . Table 4 shows test statistics concerning the time series properties of the index for the United States, Germany and Japan. For the United States and Germany the ARCH effect is strong, normality is rejected, and some serial correlation in the index is present, indicating that the ARCH-M might be appropriate. For Japan the ARCH effect is much weaker. Normality is rejected at the 5% level but not at the 1% level, and there is no autocorrelation. Hence, for Japan these LM-tests imply constant risk premia (consistent with the expectations hypothesis of the term structure).

The results in Table 5<sup>15</sup> confirm these conjectures. The point estimates of the degree of relative risk aversion ( $\delta$ ) are similar for all

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<sup>14</sup>See Pagan and Ullah (1986) for a critical analysis of the use of risk terms in regressions.

<sup>15</sup>Maximum likelihood estimates were obtained using the optimization algorithm of Berndt, Hall, Hall and Hausman (1974).



three countries. The case for time varying risk premia is strongest in the United States, while the Japanese term structure seems perfectly flat.

Figures 6 to 8 show the time series for the conditional variance. For the United States and Germany, the variance was extremely high in the period 1980 to 1982 relative to the rest of the sample. Consequently risk premia were also estimated high in that period.

Engle and Bollerslev (1986) introduce the concept of integrated ARCH processes, analogous to the literature on integrated time series, in order to describe persistence in the variance process. If in our model  $\alpha_1 + \alpha_2 = 1$ , the ARCH model has a unit root. The implication of a unit root in the ARCH model is that the unconditional variance of  $\{e_t\}$  is indeterminate, and that changes in the conditional variance dominate the movements in excess holding-period returns. If the ARCH process for the index is indeed integrated, we can get additional support for the one-factor model used to construct the index. The variance of the index  $z_t$  will dominate the covariance matrix of excess returns if  $h_t$  is integrated, but the factor model residuals  $u_t$  are not integrated (or have constant variance).<sup>16</sup> Although we have not formally tested the hypothesis of a unit root in variance, the sum of  $\alpha_1$  and  $\alpha_2$  suggests that it is present for the United States and Germany (U.S.:  $\alpha_1 + \alpha_2 = 0.97$ ; Germany  $\alpha_1 + \alpha_2 = 0.95$ ).<sup>17</sup>

## TIME VARYING RISK PREMIA AND THE SHORT-TERM INTEREST RATE

The ARCH-M model is not entirely satisfactory, since it does not identify the macroeconomic sources of risk. The ARCH-M model relates risk premia to the variance of unexpected shocks, but the nature of these shocks remains in the dark. The specification of the conditional variance model could be improved when direct observations are available for part of the common factor noise  $e_t$ .

Probably the most important variable that carries news about the term structure is the short-term interest rate. If the short-term interest rises unexpectedly and this shock is expected to persist, (for instance, if

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<sup>16</sup>This case is called cointegration in variance by Engle (1987). Testing for cointegration in variance and respecifying the index model in this framework is beyond the scope of this paper.

<sup>17</sup>Poterba and Summers (1986) find, however that the volatility of stock prices is not persistent. Different indirect evidence that a one-factor model captures the essence of the term structure is provided by the cointegration tests of Campbell and Shiller (1986) and Stock and Watson (1987).

TABLE 5

## ARCH-M Models of the Term Structure

$$z_t = c + \delta h_t + e_t$$

$$h_t = E_{t-1}(e_t^2) = \alpha_0 + \alpha_1 h_{t-1} + \alpha_2 e_{t-1}^2$$

Country	c	$\delta$	$\alpha_0$	$\alpha_1$	$\alpha_2$	$\sigma$	T
United States	-0.12 (1.1)	0.16 (2.2)	0.08 (1.9)	0.72 (9.4)	0.25 (3.5)	1.65	156
Germany	0.29 (1.6)	-0.05 (0.3)	0.05 (1.4)	0.84 (13.0)	0.12 (3.5)	1.13	161
Germany	0 --	0.16 (2.1)	0.07 (1.3)	0.84 (11.0)	0.11 (2.6)	1.11	161
Germany	0.24 (2.9)	0 --	0.05 (1.3)	0.84 (12.6)	0.12 (3.5)	1.13	161
Japan	0.01 (0.0)	0.14 (0.3)	0.30 (0.9)	0.61 (1.6)	0.13 (1.0)	1.14	109
Japan	0 --	0.15 (1.6)	0.30 (0.9)	0.61 (1.6)	0.13 (1.1)	1.08	109
Japan	0 --	0 --	0.34 (1.0)	0.58 (1.5)	0.14 (1.1)	1.10	109

|t|-values between parentheses

$\sigma$  : unconditional standard error of the residuals

United States: sample period 73:1 - 85:12

Germany: sample period 73:2 - 86:6

Japan: sample period 77:5 - 86:5

Unit of measurement is percent per month

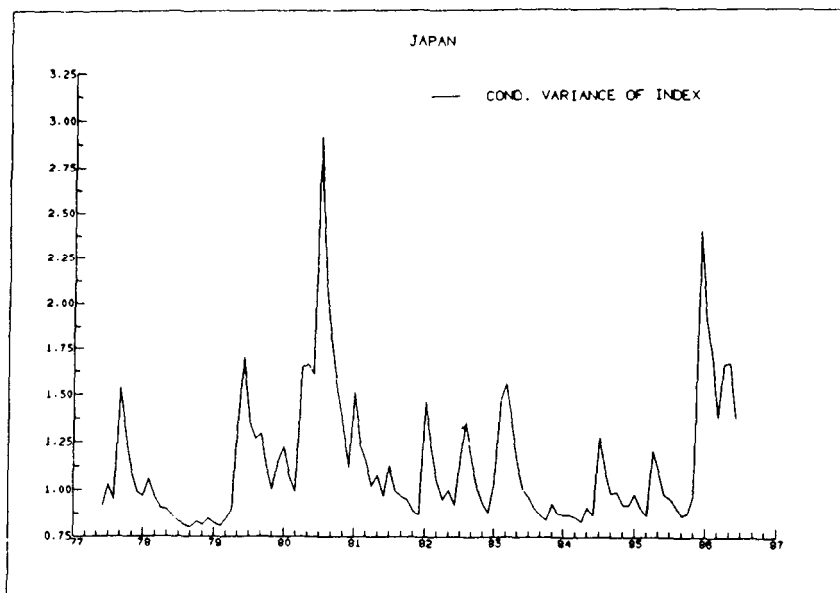


Figure 7

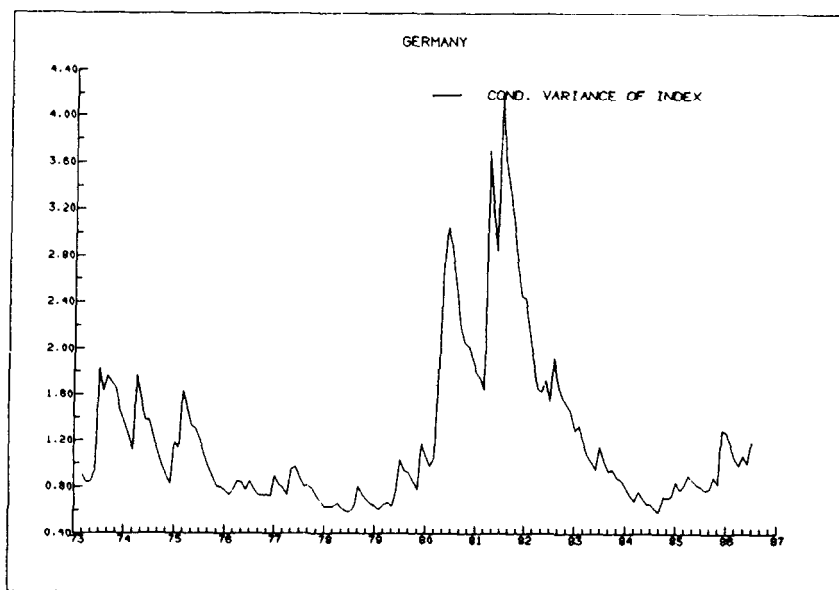


Figure 8

the short rate looks like a random walk), the expectations model of the term structure predicts that long rates will also rise. A rise of long-term rates causes a capital loss now, which gives a negative impulse to the holding-period yield in the period the shock occurs. Hence, we expect that news of the short-term interest rate has a negative impact on the index. This news variable identifies part of the risk  $e_t$  in the index:

$$z_t = \lambda_t + \beta u_t^i + \epsilon_t \quad (22)$$

where  $u_t^i$  is the unexpected shock to the nominal short-term interest rate. Again we will relate the risk premium  $\lambda_t$  to the variance of unexpected shocks. Since by construction the innovations in the short rate and the error term are uncorrelated, the variance of the full risk term ( $\beta u_t^i + \epsilon_t$ ) is a linear combination of the two components:

$$\lambda_t = c + \delta_1 h_t + \delta_2 s_t \quad (23)$$

where  $s_t$  is the conditional variance of the short-term interest rate. To implement the model we need a time series for the short-term rate innovations and the conditional variance of the short rate. We have used a simple AR(2) model with ARCH errors:

$$i_t = \gamma_0 + \gamma_1 i_{t-1} + \gamma_2 i_{t-2} + u_t^i \quad (24)$$

$$s_t = \phi_0 + \phi_1 s_{t-1} + \phi_2 (u_{t-1}^i)^2 + \phi_3 (i_{t+1|t} - \bar{i}) \quad (25)$$

where:

- $i$  : short-term (1 month) nominal interest rate
- $\bar{i}$  : sample mean of  $\{i_t\}$
- $i_{t+1|t}$  : expected level of  $i_{t+1}$  at time  $t$ .

The term  $\phi_3(i_{t+1|t} - \bar{i})$  was added to the ARCH specification to capture the empirical regularity that a high volatility of the short-term interest rate is correlated with a high level. The estimated models for the short rate in the three countries are reported in Table 6. The coefficient  $\phi_3$  is significant for all three countries. The coefficient  $\phi_3$  is significant for all three countries.

Table 7 contains the results for term-structure index. Innovations in the short rate have the correct sign and prove to be very significant in the United States and Germany, and marginally so in Japan. The results for

TABLE 6  
News and uncertainty of the short-term interest rate

$$i_t = \gamma_0 + \gamma_1 i_{t-1} + \gamma_2 i_{t-2} + u_t$$

$$s_t = E_{t-1}(u_t^2) = \phi_0 + \phi_1 s_{t-1} + \phi_2 u_{t-1}^2 + \phi_3 (i_t | i_{t-1} - \bar{i})$$

country	$\gamma_0$	$\gamma_1$	$\gamma_2$	$\phi_0$	$\phi_1$	$\phi_2$	$\phi_3$	$\sigma$
United States	0.43 (2.4)	0.89 (9.1)	0.06 (0.6)	0.18 (2.4)	0.67 (7.5)	0.20 (2.6)	0.06	1.38
Germany	0.24 (1.8)	0.80 (22.0)	0.15 (4.9)	0.45 (15.3)	0 --	0.32 (2.1)	0.12 (10.9)	0.64
Germany	0.19 (0.9)	0.83 (12.7)	0.13 (1.9)	0 --	0.98 --	0.02 (15.4)	0 --	0.63
Japan	0.68 (4.3)	0.86 (13.7)	0.03 (0.5)	0.19 (5.9)	0 --	0.59 (3.9)	0.05 (2.1)	0.34

$i$  : nominal short-term interest rate ( $\bar{i}$  is the sample mean of  $i$ )  
 $\sigma$  : unconditional standard error

TABLE 7  
The relation between the term structure and the short-term interest rate

$$z_t = c + \delta_1 h_t + \delta_2 s_t + \beta u_t + e_t$$

$$h_t = E_{t-1}(e_t^2) = \alpha_0 + \alpha_1 h_{t-1} + \alpha_2 e_{t-1}^2 + \alpha_3 (i_t | i_{t-1} - \bar{i})$$

country	c	$\delta_1$	$\delta_2$	$\beta$	$\alpha_0$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\sigma$
United States	-0.13 (1.2)	0.25 (1.1)	-0.02 (0.1)	-0.64 (9.0)	0.07 (1.7)	0.90 (15.1)	0.08 (0.7)	0.02 (2.2)	0.95
United States	-0.10 (0.9)	0.19 (1.3)	0 --	-0.65 (10.3)	0.07 (1.8)	0.92 (21.3)	0 --	0.02 (2.5)	0.96
Germany	0.21 (1.5)	0.01 (0.0)	-0.07 (0.5)	-0.50 (4.7)	0.94 (5.9)	0 --	0.11 (1.1)	0.18 (3.1)	1.08
Germany	0.19 (2.6)	0 --	0 --	-0.50 (4.9)	0.96 (6.0)	0 --	0.10 (1.1)	0.18 (3.9)	1.09
Japan	0.19 (0.7)	0.12 (0.5)	-0.53 (4.7)	-0.22 (1.0)	0.83 (5.11)	0 --	0.35 (1.7)	0.15 (1.2)	1.12

Note: The parameter  $\alpha_1$  was restricted to zero for Japan and Germany, since the unrestricted ARCH process was unstable.

United States sample period 73:1 - 85:12

Germany sample period 73:2 - 86:6

Japan sample period 77:5 - 86:5

the risk premium part of the model are not encouraging. The relation between the risk premium and the conditional variance has now disappeared completely.<sup>18</sup> The ARCH effect is clearly present in the short rate but it has no effect on the risk premium. Due to the significance of the news term  $u_t^i$ , however, likelihood ratio tests reject the simple ARCH-M models in favor of the extended model in Table 7.

## A GENERAL EXPLORATORY MODEL

The proxies for the risk premium in the third set of results include the covariance between the index and the return on short-term investments abroad ( $\text{cov}(z, \Delta e)$ ) corresponding to the theoretical model of Section III. A time series for this covariance term was prepared with the Multi State Kalman Filter (MSKF) method. An adaptive estimate of the covariance between two time series  $\{x_t\}$  and  $\{y_t\}$  is constructed from the series of cross products  $\{x_t y_t\}$ . We use this series as input variable for the MSKF algorithm and compute the underlying (permanent) level of the series as an empirical proxy for the covariance.

Before turning to the empirical results we ought to discuss two variables that were not included: yield spreads and stock returns. Yield spreads are not included because they are more or less a proxy for risk premia. If yield spreads enter the regression, a natural further question would be "What determines the risk premium in yield spreads?" With yield spreads in the regression equation, it would be hopeless to identify the macroeconomic determinants of risk premia.

Stock returns are excluded for another reason. In this paper we are concerned with risk premia in the term structure of interest rates. Stock returns also carry a risk premium, but we think the risk premium in stock returns is more industry specific and can certainly not be described by the one-factor model used here. On the contrary, we prefer to see our term-structure index as one factor in stock returns.<sup>19</sup> We experimented with including stocks in the first stage of the model, the construction of an index. It then appeared that the portfolio weight for stocks was virtually zero due to a high specific variance (estimated in the preliminary factor

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<sup>18</sup>Experimentation with different models for the short-term interest rate did not improve the results.

<sup>19</sup>The relation between stock returns and the term structure is further empirically investigated within a latent variable model by Campbell (1987).

model). For Germany and United States its risk sensitivity  $b_j$  was larger, however, than that on the longest bonds.

Besides the proxy for the risk premium, the results in Tables 8 to 10 also include a selection of macroeconomic new variables. The first regression in each table includes all explanatory variables in the information set; the third column has our preferred OLS specification.

For the United States innovations in the short-term interest rate, unexpected money growth, and unexpected inflation all identify part of the news component of the index. The residuals still show signs of ARCH. The innovations series do not capture all the heteroskedasticity in bond returns. Some important news variable is still missing. Especially the enormous volatility in excess returns between 1980 and 1982 is not explained by the innovations in inflation, money, and nominal interest rates. The variables that should identify the risk premium are all insignificant.

We tested specifically for effects of the expected longer-term growth in real balances. This variable, often connected to the slope of the yield curve in popular discussions, enters with a significant positive sign. A higher yield spread is correlated with an expansionary monetary policy (see the second and last regressions in Table 8). This correlation, however, is absent in Germany; in Japan the same variable has a negative sign.

The results for Germany are largely identical, with the exception that we also included variables related to the United States, assuming that economic developments in the U.S. are (weakly) exogenous with respect to the rest of the world. The German term-structure index is strongly related to the index for the U.S. and to changes in the exchange rate. These are included here as the ex post excess return for U.S. investors when investing in German short-term bonds. This variable contains both news and a risk premium. Its inclusion is motivated by the simple theoretical mean variance model of Section III. The variable  $(i^{WG} - i^{US} - \Delta e)$  corresponds to the difference between the logarithm of the spot and the lagged forward exchange rate. The only domestic variable that remains significant is the innovation in the short-term interest rate.

There is no evidence of ARCH in the error term anymore. This is due to the inclusion of the U.S. term-structure index. The heteroskedasticity in the German term-structure index is fully matched by the heteroskedasticity in U.S. term structure. Since both are probably integrated in variance, this provides a case for setting up a two-country model in which a single factor simultaneously describes the dominant movements in the U.S. and

TABLE 8

Term-Structure Models United States  
Dependent Variable: Term Structure Index  $z_t^{us}$

constant	0.17 [0.3]	0.12 [1.1]	0.13 [1.6] (1.5)
$i^u$	-0.67 [7.5]		-0.67 [7.3] (10.0)
$p^u$	0.54 [1.9]		0.60 [2.3] (2.2)
$m^u$	0.42 [2.8]		0.36 [2.6] (2.9)
$i^e$	-0.88 [1.5]		
$(\Delta p)_{18}^e$	-0.05 [0.7]		
$(\Delta m - \Delta p)_{18}^e$	0.11 [1.2]	0.05 [2.4]	0.04 [2.2] (1.7)
var(i)	0.11 [1.3]		
sample	73:2 - 85:3	73:2 - 85:3	73:2 - 85:3
$R^2$	0.49	0.02	0.48
DW	2.12	1.57	2.09
ARCH(1)	4.23	8.90	3.45
CHOW	2.88	0.81	4.11

$i^u$  unexpected changes in short-term interest rate (see table 5.2)

$p^u$  unexpected inflation (computed by MSKF)

$m^u$  unexpected money growth (computed by MSKF)

$i^e$  expected level of short-term interest rate (see table 5.2)

$(\Delta p)_{18}^e$  expected long-term inflation (computed by MSKF)

$(\Delta m - \Delta p)_{18}^e$  expected long-term real money growth (computed by MSKF)

var(i) variance of short-term interest rate (see table 5.2)

CHOW Test for parameter constancy, sample split in 79:9

ARCH(1) Test for heteroskedasticity (see Table 4.4)

(.) |t|-statistic

[.] |t|-statistic computed by White's (1980) heteroskedastic

consistent estimator of standard errors. These t-statistics are reported whenever the ARCH statistic was significant at the 5% level.



TABLE 9

Term Structure Models Germany  
 Dependent Variable: Term Structure Index  $z_t^{WG}$

constant	0.62 (1.2)	0.002 (0.1)	0.14 (2.0)
$i^u$	-0.56 (5.2)		-0.48 (5.0)
$p^u$	0.01 (0.1)		
$m^u$	-0.05 (0.7)		
$i^e$	-0.03 (1.0)	0.05	
$(\Delta p)_{18}^e$	-0.01 (0.2)		
$(\Delta m - \Delta p)_{18}^e$	-0.03 (0.7)	0.05 [1.4]	
$z^{us}$	0.36 (6.2)		0.36 (6.6)
$i - i^{us} - \Delta e$	0.09 (4.5)		0.07 (3.5)
$cov(z, \Delta e)$	14.3 (0.2)		
$var(i)$	0.06 (0.7)		
sample	74:4 - 85:3	73:7 - 85:12	73:7 - 85:12
$R^2$	0.52	0.01	0.43
DW	1.90	1.54	2.01
ARCH(1)	0.10	7.71	0.15
CHOW	2.95	0.43	5.26

See Table 11 for meaning of explanatory variables. Additional variables:

$z^{us}$  : term structure index United States

$i - i^{us} - \Delta e$  : excess return for U.S. investors on short-term German investments

$cov(z, \Delta e)$  : covariance between  $z^{WG}$  and  $(i - i^{us} - \Delta e)$  (computed by MSKF)

TABLE 10  
Term Structure Models Japan  
Dependent Variable: Term Structure Index  $z_t^J$

constant	0.73 (1.3)	0.27 (2.1)	0.06 (0.7)	0.29 (2.4)
$i^u$	-0.35 (2.6)		-0.35 (2.3)	-0.39 (2.7)
$p^u$	0.14 (1.3)		0.31 (2.6)	0.22 (1.9)
$m^u$	0.03 (0.8)			
$i^e$	-0.84 (0.9)			
$(\Delta p)_{18}^e$	-0.19 (2.4)			
$(\Delta m - \Delta p)_{18}^e$	-0.10 (3.0)	-0.06 (1.8)		-0.06 (2.1)
$z^{us}$	0.27 (4.3)		0.19 (3.2)	0.12 (1.9)
$i - i^{us} - \Delta e$	0.10 (4.5)		0.09 (3.8)	0.11 (4.4)
$cov(z, \Delta e)$	-25.1 (0.6)		72.9 (2.4)	0.23 (0.0)
$var(i)$	-0.51 (3.5)			
sample	77:5 - 85:3	77:5 - 85:3	77:5 - 85:12	77:5 - 85:3
$R^2$	0.48	0.03	0.35	0.37
DW	2.15	1.82	1.95	2.18
ARCH(1)	0.12	2.16	0.71	0.96
CHOW	1.53	1.70	1.89	1.98

see Table 10 for meaning of explanatory variables. Additional variables:

$z^{us}$  : term structure index United States

$i - i^{us} - \Delta e$  : excess return for U.S. investors on short-term Japanese investments.

$cov(z, \Delta e)$  : covariance between  $z^J$  and  $(i - i^{us} - \Delta e)$  (computed by MSKF)

German term structure.

For Japan we used the Japanese analogs of the regressors for Germany. The regression results are less clear as it proved difficult to obtain an econometrically acceptable data description with stable, constant parameters. Only two variables stand out as being significant over the full sample period: innovations in the nominal short-term Japanese interest rate and in inflation.

## RECURSIVE ESTIMATES

Since the estimated parameters are sensitive to the sample period, and a CHOW test frequently rejects parameter constancy in all three countries, we wanted to monitor how coefficients changed over time. The best starting point for such an investigation is the recursive ordinary least squares method. This algorithm requires an initial block of data to compute initial estimates of the coefficients and the matrix  $(X'X)^{-1}$ , with  $X$  the matrix of the regressors. The algorithm processes each observation in a recursive manner to update the parameter estimates. At the end of the period of estimation, the final estimate for the coefficient vector will be identical to the OLS estimate for the full sample period.<sup>20</sup>

Recursive least squares with a possible correction for the effects of outliers could be applied in a setting in which the true value of the model parameters were constant but had to be discovered by the economic agents. We have experimented also with two algorithms that are specifically designed to allow for variation in the model parameters: discounted recursive least squares and a multivariate Kalman filter.<sup>21</sup> Discounted recursive least squares artificially increases the so-called gain factor in order to give more weight to the most recent observation and to discount the past.

The multivariate Kalman filter method is based on the book by Ljung and Söderström (1983). Now, certain elements in the matrix  $(X'X)^{-1}$  are artificially increased after each adjustment of the parameters. We have limited our experiments to adding small constants to the elements of the

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<sup>20</sup>The residuals are not identical to the OLS residuals. Goodwin and Payne (1977) and Ljung and Söderström are excellent references for recursive least squares methods and for the correspondence between OLS, recursive least squares, and multivariate Kalman filters.

<sup>21</sup>Zellner (1986) discusses the option of using a variable parameter algorithm in the absence of a well-specified theory about the causes of changes in the parameters.

main diagonal of the matrix that corresponds to  $(X'X)^{-1}$  in the OLS case.<sup>22</sup>

We have re-estimated the equations in the second and third columns of Tables 8 to 10 using a number of recursive algorithms. Results using recursive ordinary least squares (ROLS) are reported here, since they serve as a noncontroversial test of the hypothesis that regression coefficients in the OLS model are constant over time.<sup>23</sup>

The results differ between the United States and the other two economies. In the U.S. (where we did not insert an exchange-rate variable or a foreign bond index), the real balance effect changed over time. The coefficient on this variable disappears in late 1978 and early 1979 and reappears in September 1982. All coefficients are especially volatile in late 1979 and the first half of 1980, which was a period with many outliers. The residual variance exhibits a permanent jump in October 1979.

For Germany and Japan the most striking feature of the ROLS regressions is the pattern of the coefficients on the variables that are related to returns in other markets than the domestic bond market. In Germany, the U.S. term-structure index becomes significant from 1980:1 onwards. The exchange-rate variable exhibits a permanent jump at the same time and becomes significantly positive: unexpected appreciations of the dollar are correlated with low returns on German bonds and vice versa. This applies to Japan, too: a stronger dollar lowers Japanese bond returns. The equation for Japan also includes a covariance term that was significant in the OLS specification. We show its pattern over time in Figure 9. Figure 10 shows the coefficient on the U.S. index in the equation for Japanese bond returns. Both parameters exhibit considerable variation over time.

Some sharp discontinuities in the coefficients coincide with changes in domestic or foreign monetary policy. For Japan, the recursive estimates highlight a discontinuity in all the parameters in May 1980. The rate of inflation in Japan reached a peak in this month of 18.4% at an annual rate (W.P.I.). Suzuki (1980) explains: "After the second oil crisis, the

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<sup>22</sup>Bomhoff (1987) has further discussion and an application to reduced-form equations for equation rates.

<sup>23</sup>Results using discounted recursive least squares, recursive least squares with a cut-off point for outliers, and a multivariate Kalman filter were qualitatively similar to the ROLS results and are obtainable from the authors on request. The recursive algorithms will also be made available.

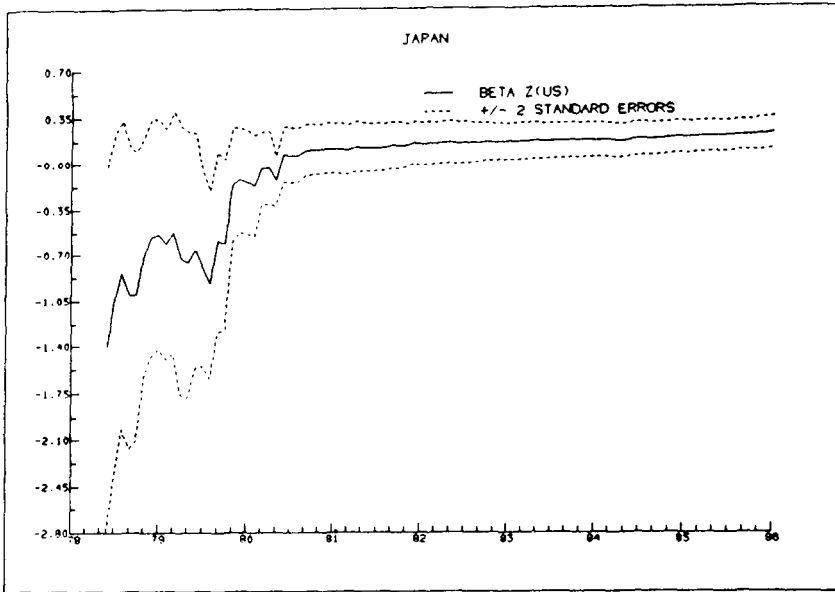


Figure 9

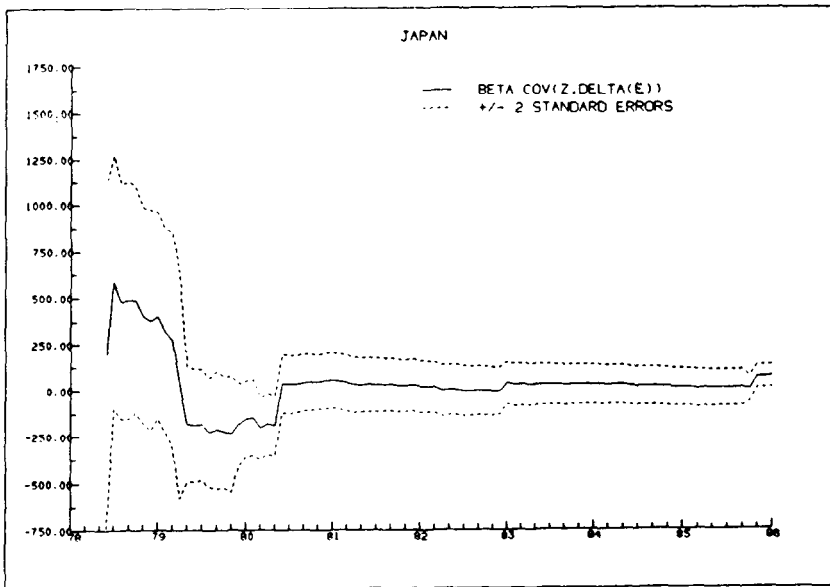


Figure 10

official discount rate and regulated rates including the deposit rates were quickly raised to levels comparable to those at the last peak of the first crisis; they then were lowered quickly as inflation subsided" (p.71).

There is a break in the coefficient for  $\text{cov}(z, \Delta e)$  in November 1985. This month is mentioned by Ito (1986) as the time of a significant change in monetary policy in Japan. In the German interest rate data, 1982:2 stands out as an important outlier. This month witnessed a sharp peak in short-term German interest rates, caused by uncertainty about the mark in the European Monetary System.<sup>24</sup>

## VI. CONCLUSIONS

Some conclusions from the analysis or, better perhaps, some suggestions for further work, follow.

1. Although there are dominant common movements in the return on long-term bonds in the United States, Germany, and Japan, the macroeconomic determinants of the term structure of interest rates differ across countries. For example, movements in the expected rate of growth in real money balances are easily correlated with movements in the risk premium in the United States bond yields. A more expansionary monetary policy results in a higher risk premium. In Germany and Japan the same measure of the expected rate of growth of real balances does not exhibit the same comovement with the risk premium on long-term bonds.

2. The slope of the United States yield curve has an effect on the slope of the yield curve in Japan and Germany. However, the size and direction of this effect are not constant over time. An international asset pricing model should leave room for time-varying channels of influence between the United States and the other countries. This may be hard to achieve in a dynamic optimizing model for more than one country. Perhaps further partial analyses are required in order to gain more understanding of patterns in bond returns.

3. Risk premia change over time, and the data suggest that contemporaneous changes in the measured variance of bond returns are not

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<sup>24</sup>We note that interest rates in other members of the European Monetary System exhibit more frequent temporary spikes in short-term interest rates. A simple autoregressive process is inadequate to capture the dynamics of the short rate in these economies.

the principal cause. Empirical work will need to incorporate changing covariances between bond returns and returns on other domestic and foreign assets. Or statistical analyses ought to allow for more complicated dynamics in risk premia. Perhaps uncertainty about returns increases risk premia for some time beyond (or before) the period of high volatility in measured returns [see Flood and Hodrick (1986) for further discussion].

4. The Federal Reserve Board does not have a monopoly on the causation of outliers in observed returns on financial assets. Returns on bonds in all three countries exhibit strong deviations from normality, and a statistical analysis should allow for this in a more serious way than a simple shift dummy for whatever happened in November 1979.

## DATA APPENDIX

The data used for the regressions in Section V will be made available to interested researchers. Please mail a formatted floppy disk to the authors.

### 1. Bond data

U.S.: Maturity classes are as in Bodie, Kane, and McDonald (1983). Data through 1983 were kindly provided by Alex Kane. The remaining data are computed from the University of Chicago's CRSP Bond file, taking average holding-period returns for all bonds near par in each maturity class.

Japan: Data was kindly provided by Mr. H. Shirakawa of the Bank of Japan. Monthly returns are computed as averages for at least 10 bonds in each class. The more recent starting point of the Japanese data is dictated by an institutional change in April 1977 when restrictions on the sale of government bonds by underwriting syndicates were eased considerably (Shikano, 1985).

Germany: Bond prices from the "Frankfurter Allgemeine." for each of the four maturity classes all federal bonds were used. Monthly returns were weighted with the amounts outstanding as indicated by the Annual "Geschäftsbericht der Bundesbank."

Figures 1 and 2 show the yield curve for Germany and Japan during a five-year period that includes most outliers in the returns.

### 2. Short-term rates

U.S.: For 1984-1985 the one-month interest rate is not computed from bond returns but as the one-month CD rate plus the risk spread between 3-month CD's and 3-month Treasury bills.

Japan: Domestic short-term interest rates at 1, 2, and 3 months are used.

Germany: One-month Interbank rate.

### 3. Exchange rates

End-of month bilateral exchange rates were kindly provided by the Federal Reserve Bank of St. Louis. To maintain compatibility with the model, the internal (domestic) one-month interest rates were used to compute the unexpected change in the exchange rate.

4. Other macroeconomic data: all taken from standard sources, mostly the I.F.S. tape.



## REFERENCES

- Berndt, E.K., Hall, B.H., Hall, R.E. and Hausman, J.A.  
(1974) Estimation and Inference in Nonlinear Structural Models, *Annals of Social and Economic Measurement*, 3: 551-614.
- Bodie, Z, Kane, A. and McDonald, R.  
(1983) Why are Real Interest Rates so High?, *National Bureau of Economic Research, Working Paper Series*, no. 1141.
- Bollerslev, T., Engle, R.F. and Wooldridge, J.M.  
(1985) A Capital Asset Pricing Model with Time Varying Covariances, UCSD Discussion Paper no. 85-32.
- Bomhoff, E.J.  
(1982) Predicting the Price Level in a World that Changes all the Time, K. Brunner and A.H. Meltzer (eds.), *Economic Policy in a World of Change*, Carnegie-Rochester Conference Series on Public Policy, 17: 7-38. Amsterdam: North Holland.
- 
- (1983) Monetary Uncertainty, North-Holland, Amsterdam.
- 
- (1987) The Dollar-Yen Exchange Rate, forthcoming, Bank of Japan Monetary and Economic Studies.
- 
- and Korteweg, P.  
(1983) Exchange Rate Variability and Monetary Policy under Rational Expectations: Some Euro-American Experience 1973-79. *Journal of Monetary Economics*, 11: 169-206.
- Brown, S.J. and Weinstein, M. I.  
(1983) A New Approach to Testing Asset Pricing Models: the Bilinear Paradigm, *Journal of Finance*, 38: 711-743.

Campbell, J.Y.

- (1987) Stock Returns and the Term Structure, *Journal of Financial Economics*, 18: 373-399.

---

and Shiller, R.J.

- (1986) Cointegration and Tests of Present Value Models, Cowles Foundation Discussion Paper no. 785.

Chan, K.C. and Stulz, R.M.

- (1986) Risk and the Economy: A Finance Perspective, unpublished paper, Ohio State University.

Clarida, R.H. and Campbell, J.Y.

- (1987) The Term Structure of Euromarket Interest Rates: An Empirical Investigation, *Journal of Monetary Economics*, 19: 25-44.

Conroy, R.M. and Rendlemen, R.J.

- (1983) Pricing Commodities When Both Price and Output Are Uncertain, *The Journal of Futures Markets*, 3: 439-451.

Engle, R.F.

- (1982) Autoregressive Conditional Heteroskedasticity With Estimates of the Variance of United Kingdom Inflation, *Econometrica*, 50: 987-1007.

- 
- (1987) Multivariate ARCH With Factor Structures-Cointegration in Variance, Manuscript, University of California, San Diego.

---

and Bollerslev, T.

- (1986) Modelling the Persistence of Conditional Variances, *Econometric Review*, 5: 1-50.

---

and Granger, C.W.J.

- (1987) Co-Integration and Error Correction: Representation, Estimation and Testing, *Econometrica*, 55: 251-276.

Engle, R.F., Lilien, D.M. and Robins, R.P.

- (1987) Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model, *Econometrica*, 55: 391-407.

Fama, E.F.

- (1970) Multiperiod Consumption-Investment Decisions, *American Economic Review*, 60: 163-174.

Flood, R.P. and Hodrick, R.J.

- (1986) Asset Price Volatility, Bubbles and Process Switching: *Journal of Finance*, 41: 831-842.

Goodwin, G.C. and Payne, R.L.

- (1977) Dynamic System Identification: Experiment Design and Data Analysis. New York: Academic Press.

Gultekin, N.B. and Rogalski, R.J.

- (1977) Government Bond Returns, Measurement of Interest Rate Risk, and the Arbitrage Pricing Theory, *Journal of Finance*, 40: 43-61.

Hakkio, C.S. and Leiderman, L.

- (1986) Intertemporal Asset Pricing and the Term Structures of Exchange Rates and Interest Rates, *European Economic Review*, 30: 325-344.

Hansen, L.P. and Singleton, K.J.

- (1983) Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns, *Journal of Political Economy*, 91: 249-265.

Ito, T.

- (1986) The Intra-Daily Exchange Rate Dynamics and Monetary Policies after the G5 Agreement, paper presented at the International Symposium on "Current Theoretical and Policy Issues in the U.S. and Japanese Economy," Tokyo.

Jarque, C. and Bera, A.K.

- (1980) Efficient Tests for Normality, Homoskedasticity, and Serial Independence of Regression Residuals, *Economics Letters*, 6: 255-259.

Kool, C.J.M.

- (1982) The Multi-State Kalman Filter Method, K. Brunner and A.H. Meltzer (eds.), *Economic Policy in a World of Change*, Carnegie-Rochester Conference Series on Public Policy, 17: 39-46. Amsterdam: North Holland.

Ljung, L. and Söderström, T.

- (1983) Theory and Practice of Recursive Identification. Cambridge, Massachusetts: The MIT Press.

Long, J.B. Jr.

- (1974) Stock Prices, Inflation and the Term Structure of Interest Rates, *Journal of Financial Economics*, 1: 131-170.

Malliari, A.G. and Brock, W.A.

- (1981) Stochastic Methods in Economics and Finance, Amsterdam: North Holland.

Mardia, K.V., Kent, J.T. and Bibby, J.M.

- (1979) Multivariate Analysis, London: Academic Press.

Pagan, A. and Ullah, A.

- (1986) The Econometric Analysis of Models with Risk Terms, unpublished paper, Australian National University.

Poterba, J.M. and Summers, L.H.

- (1986) The Persistence of Volatility and Stock Market Fluctuations, *American Economic Review*, 76: 1142-1151.

Ross, S.A.

- (1976) The Arbitrage Pricing Theory of Capital Asset Pricing, *Journal of Economic Theory*, 13: 341-360.

Stock, J.H. and Watson, M.W.

(1987) Testing for Common Trends, unpublished discussion paper.

Suzuki, Y.

(1986) Money, Finance and Macroeconomic Performance in Japan, New Haven and London, Yale University Press.

White, H.

(1980) A Heteroscedasticity-Consistent Covariance Matrix Estimator and A Direct Test for Heteroscedasticity, *Econometrica*, 48: 817-838.

Zellner, A.

(1986) Discussion of the Chan-Stulz Paper, unpublished paper, University of Chicago.