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         Stability of Velocity in the Group of Seven Countries:
                        A Kalman Filter Approach

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                                  Abstract

    This paper estimates forecasting models using annual data for the
income velocity of money in the G-7 countries.  The predictions are
conditional upon the realized value of the long-term domestic government
bond rate.  Such conditional forecasts did not deteriorate over the period
1980-1988 as compared with the earlier postwar period.  Velocity of M1 is
found to be very interest-elastic in almost all countries; velocity of M2
less so.  The specifications (based on Kalman filters and smoothers) point
to a non-constant (stochastic) trend in velocity, hence questioning the
assumptions required for the cointegration techniques used in other research
on the demand for money.

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## I. Introduction

In the early 1980s, many economists became convinced that the demand for money schedule was too unstable to be used for policy purposes. One reason was the influential article by Cooley and LeRoy (1981) which cast serious doubts on the identification of a demand for money function. Another cause was the apparent failure of monetary models to explain movements in floating exchange rates, in particular changes in the external value of the U.S. dollar. Also, many demand for money relations for U.S. M1 or M2 appeared to break down when used for post-sample forecasting, particularly those models that incorporated a small or zero interest rate elasticity. Finally, domestic financial deregulation or international currency substitution were claimed to have shifted the demand for money in an unpredictable manner.

Prescriptions for monetary policy that are formulated in terms of a path for some monetary aggregate must be based on a demand for money function. Doubts about the stability of that function generate doubts about such recipes for policy. This is one reason for the interest in policy prescriptions that are based on targets for interest rates or exchange rates, because such policy rules can (under sometimes unattractive assumptions) be derived from macroeconomic models that do not require identification of a demand for money schedule, or precise knowledge about the interest rate or income elasticities of the demand for money.

Statements about the stability or otherwise of the relationship between money and nominal income are conditional upon the selection of countries in the data set and on the type of statistical analysis performed. Here I have applied identical specifications to annual postwar data for all Group of Seven (G-7) countries, using both M1 and M2. The statistical methodology in the paper reflects an important difference of opinion regarding the demand for money function.

Some researchers do not reject the hypothesis that the levels (of the natural logarithms) of money, income and possibly a relevant interest rate are cointegrated, meaning that a regression of the level of real balances on the level of income (and the opportunity cost variable) is permissible (see Boughton (1990), who uses data from five of the G-7 countries; see also Hendry and Ericsson (1990) for the United Kingdom only, and Hoffman and Rasche (1989) for the United States). 1/ Others prefer to work in terms of first differences of money, income and interest rates without reliance on a long-term relationship in terms of the levels (see, for example, Rasche (1987), and Hetzel and Mehra (1989) for the United States). Finally, the monograph by Bordo and Jonung (1987) on the long-run behavior of velocity in many countries shows that velocity has a stochastic trend. Unless explanatory variables can explain all changes in the rate of growth of

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1/ When no danger of confusion exists, the words "natural logarithm of" will be omitted in the sequel.

velocity--and the work by Bordo and Jonung suggests that neither income nor institutional variables that represent monetization or economic development can provide more than a partial explanation--it follows that regressions in first differences are misspecified: one would have to difference at least twice.

The simple fact that there are three coexisting schools of thought on this particular issue proves how hard it is to resolve the dispute with least squares regression techniques. Recall that the natural context for any least squares model is that of stationary variables, because least squares regressions for nonstationary variables have to work with a system matrix X'X that is a function of the number of data points. Such regressions do not satisfy ergodicity, meaning that it is not plausible that a single collection of historical data can be used for the estimation of coefficients with distributions that relate to repeated sampling. 1/ Of course each differencing operation increases the probability that the transformed series are stationary. But, if the relationship when specified in terms of levels is subject to both temporary and permanent disturbances, differencing results in a deterioration of the signal-to-noise ratio and less well-determined coefficients.

By contrast to linear regression techniques, Kalman filters and smoothers are designed to work with non-stationary data, because the filters and smoothers produce distributions of the so-called state variables that are conditional on the previous realization of the states. For that reason, non-stationarity in itself presents no problem, and ergodicity can be satisfied, implying that the distributions of the coefficients have a meaningful interpretation. The only reason why Kalman filtering has not yet become the natural way to model multivariate time series has been the technical difficulty to combine estimation of the states with estimation of other parameters required to run the filter successfully.

This paper presents a method for estimating states and parameters jointly, using smoothing algorithms developed by Maybeck (1979, 1982) together with an estimation technique developed by Dempster, Laird and Rubin (1977) and adapted to the Kalman filter case by Shumway and Stoffer (1982). The Kalman filter model will be estimated in terms of levels, with allowance for three types of shocks to velocity (V):

     (1) temporary shocks to the level of V;

     (2) permanent shocks to the level of V;

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1/ Durlauf and Phillips (1988) provide an excellent theoretical analysis of the difficulties that arise when ordinary least squares are applied to nonstationary time series with the possibility that the errors are also nonstationary and nonergodic. See also Plosser and Schwert (1979) and Nelson and Plosser (1982). This line of research originated with Paul Newbold, see Granger and Newbold (1974)

(3) permanent changes in the trend of V.

Note that type (2), permanent shocks to the level, can be described also as representing temporary disturbances to the rate of growth.

The variances of the different types of shocks and hence their relative importance will be estimated on the basis of the data. In this way the methodological difficulties associated with indirect tests for non-stationarity or cointegration are avoided: the data will tell us whether it is useful or not to account for stochastic changes in the trend. 1/

However useful for dealing with nonstationarity and mixtures of different types of shocks, the Kalman filter cannot deal with the issues raised by Cooley and LeRoy. These authors emphasized two complications that hamper empirical investigations of the demand for money schedule:

1. disentangling demand and supply of money may be impossible; 2/

2. measurement errors in the explanatory variables effect the estimated coefficients in the demand for money relation.

Perhaps the best response is to give up the ambition to _estimate_ a demand for money function and try only to _forecast_ the income velocity of money. This paper takes the position that forecasts of velocity remain useful, even though it may not be possible to classify the forecast formula as an inversion of the demand for money schedule. Thus, the forecasts may be based on some mixture of demand and supply schedules, and the coefficients will indeed be sensitive to measurement errors in the right-hand-side variables and possibly to the so-called Lucas critique. 3/ Hence, the principal connections between the forecasting formulas and economic theory are the choice of explanatory variables (legitimized by their association with the demand or perhaps the supply of money), the maximum length of any lags in the formulas, and perhaps prior distributions on some of the coefficients.

The remainder of the paper is organized as follows. Section II introduces a multivariate Kalman filter technique that can be used to estimate a relationship between the level of V, and the level of the interest rate. Section III presents the results of implementing this multivariate Kalman filter for the velocity of M1 and M2 in all G-7 countries using annual data. Section IV tests a number of simple hypotheses

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1/ See Swamy, Von zur Muehlen and Mehta (1989) for a very critical methodological discussion of cointegration tests.

2/ See, for example, Hamilton (1989) for a brief analysis of why standard money demand equations are a mixture of supply and demand effects.

3/ Neither issue can be circumvented with the use of instrumental variables (see Cooley and LeRoy).

regarding the stability of velocity and the size of the forecast errors in velocity during the 1980s.  Section V draws some statistical and economic conclusions.


## II.  A Kalman Filter Model for Velocity

Consider the simplest possible relationship between real balances, real income and an interest rate:

(1) $$p_t + y_t - M_t = V_t = c + \alpha tr_t + \theta i_t + u_t.$$

In equation (1), $p_t$ represents the natural logarithm of the price level in an economy, $y_t$ the log of a measure of income appropriate to the demand for money, $M_t$ the log of the money supply and hence $V_t$ the log of the income velocity of money.  c represents a shift term in the regression, $tr_t$ a linear trend for the log of V, $i_t$ the log of some relevant interest rate and $u_t$ the residual in the regression.  $\alpha$ and $\theta$ are coefficients to be estimated.

If one models in terms of levels, one has to accept that the residual part of the equation will be non-stationary.  Time-varying stochastics offer the best chance to cope with the dynamic aspects of the demand for money listed above.  One way to embed the linear least squares equation (1) in a richer dynamic model is to change to the state-space formulation.  The state vector is composed of all regression coefficients.  The state transition matrix would be the unit matrix in the case of recursive least squares without correction for serial correlation, but can be different in order to represent dynamic features that are hard or impossible to model in the least squares context.

The general state-space notation is as follows:

(2) $$V_t = \begin{bmatrix} 1 & 0 & i_t \end{bmatrix} \begin{bmatrix} c_t & tr_t & \theta_t \end{bmatrix}^T + u_t$$

$$\text{var} (u) = R$$

(3) $$\begin{bmatrix} c \\ tr \\ \theta \end{bmatrix}_{t+1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ tr \\ \theta \end{bmatrix}_t \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$\text{var} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} Q_1 & Q_{12} & 0 \\ Q_{12} & Q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equation (2) is the observation equation. It states that the level of the log of velocity, V, equals the sum of a shift parameter, the product of the interest rate elasticity, $\theta$, and the long-term interest rate, $i_t$, and a residual term $u_t$. This observation equation is identical to an ordinary regression equation.

The Kalman filter methodology adds equation (3), the so-called state update equation. It shows how three state variables change from period to period. The equation has a deterministic part and a stochastic part. In the deterministic part, the shift parameter is adjusted upwards in each period by the amount, $tr_t$, which represents a trend. In the stochastic part of equation (3), the trend term, $tr_t$, is subject to a stochastic shock, $w_2$, and the shift parameter is subject to permanent stochastic shocks, $w_1$. The interest rate elasticity is not subject to stochastic shocks over time.

The user of a Kalman filter is asked to provide estimates of the variances $Q_1$, $Q_2$, their covariance and R. The Kalman filter then processes the data "on line" and produces estimates of the state variables--here, the shift parameter, the trend and the interest elasticity--and their variance-covariance matrix, $P_t$.

The variances, $Q_1$, $Q_2$, their covariance ($Q_{12}$) and R may be chosen in such a way that the specification becomes equivalent to either equation (1) in terms of the levels or the same specification in terms of first or second differences. The Kalman filter specification of equations (2) and (3) thus includes both the levels and the first-difference specification. Other statistical techniques for comparing levels and first difference specifications suffer from the disadvantage that the two competing hypotheses are non-nested.

### III. Velocity in the G-7 Countries

All data are taken from International Financial Statistics. Starting points for the analysis were dictated by data availability as indicated below in Table 1; the terminal year is 1988. There are discontinuities in some of the monetary series; a dummy variable is added for each of the non-trivial breaks in a series for M1 or M2. Since estimation is in terms of levels, the dummies are of the type {0,0,...0,1,1,...1,1}. The discontinuities in the money series are as follows:

|        | M1   | M2   |
|--------|------|------|
| US     | - -  | 1959 |
| UK     | 1981 | 1975 |
|        |      | 1981 |
| France | 1958 | 1958 |
|        | 1969 | 1969 |
|        | 1977 | 1977 |
| Canada | 1968 | 1967 |
|        |      | 1968 |

The economic model is the simplest possible one.  The income elasticity of money demand is fixed at unity and a single interest rate is used to represent the opportunity cost of money, using the simplifying assumption that the own rate of return on money in each country is constant over time at the margin.  With such simple assumptions the resulting models will not be the optimal forecasting tools for velocity.  However, the results from these simple specifications may contribute more convincingly to the debate about the predictability of velocity, because uniform and simple models for seven different countries are less subject to the suspicion of being based on data mining than multi-parameter models with extensive lag structures and many free parameters that are tuned to the actual data in each country.

The only free parameters in the models are the interest elasticity which is assumed to be constant over time, and three variance terms: the variance of the permanent shocks to the level of the series, the variance of the permanent shocks to the trend in velocity and the covariance between these two types of disturbances. 1/

The exogenous explanatory variable is the domestic yield on long-term government bonds.  No experiments were undertaken with other rates of return or with lag structures, and the same specification was imposed for all countries.

The filtering and estimation algorithm consists of five different blocks.  First, there is a normal ("forward") Kalman filter that produces an estimate of the state variables at time T+1 (in our case: the shift parameter, the trend and the interest rate elasticity) based on all the data

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1/ The variance of the temporary shocks to the level of velocity could be seen as a fourth variance parameter, but the models are homogeneous of the first degree in all the variance and covariance terms. Hence, this variance is best viewed as computed ex-post from the results of the Kalman filter.

time t=1 up to and including time t=T. Second, a backward filter is used which generates a backward "forecast" for time T using all the data from period T+1 through to the final period.

A smoothed estimate of the state at time t=T can be formed by combining the forward and backward filters. In order to generate a meaningful covariance matrix for the smoothed estimates of the states, one has to start both filters with an uninformative prior distribution for the covariance matrix of the states. With this initialization, the smoothing algorithm will reproduce the OLS variance matrix of the parameters (and the OLS residuals) in the special case that all the states are constant and correspond to OLS parameters.

The fourth block of the algorithm uses the results of the Kalman smoother to compute adjustments to the unknown parameters (the four unknown variance terms), based on the Expectation Maximization algorithm, described by Dempster et al. (1977) and adapted to this case by Shumway and Stoffer (1982). 1/ Then the separate forward and backward Kalman filters (blocks 1 and 2) are run again in order to prepare inputs for the Kalman smoother in the next iteration. This process stops when the estimated values of the unknown parameters have converged to their optimal values. 2/

Finally the fifth block of the algorithm is applied just once. It starts with the optimal values for the interest rate elasticity and all variance and covariance terms and uses these inputs for a single run through the data. The forecast errors of this filter are analyzed in the tables. Such a forward filter does use a few inputs that are based on an analysis of the complete sample period: the interest rate elasticity and the relative importance of permanent shocks to the level of velocity versus permanent shocks to its growth rate. However, the final forward filter does not use knowledge about the specific realization of the shocks in the sample. Hence it should be classified as a recursive method rather than an ex-post method such as ordinary least squares or least squares with an error-correction specification.

Table 1 summarizes the results for the G-7 countries. For each country the interest rate elasticity is shown for M1-velocity and M2-velocity, together with the estimated standard error of the coefficient. All t-values are significant at the 0.05 level on a two-sided test, except in France, where the interest rate elasticity for M2 is insignificant and the coefficient for M1 has a t-value of 2.01. In all countries the interest

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1/ See Nelson (1988) for evidence from his univariate research of U.S. GNP that optimization with respect to the unknown variances of the different shocks to the level and the shocks to the trend of a nonstationary time series may be a delicate matter. This is a topic for additional research.

2/ See Bomhoff (1990) for further details on the statistical procedure used.

Table 1.  Forecast Errors of Velocity

| | M1 | | M2 | |
|---|---|---|---|---|
| | interest elasticity | standard error | interest elasticity | standard error |
| United States 1956-1988 | 0.228 (0.033) | 2.3 2.6 | 0.239 (0.035) | 2.2 2.3 |
| Japan 1968-1988 | 0.223 (0.090) | 5.1 4.3 | 0.163 (0.064) | 3.6 2.7 |
| Germany 1958-1988 | 0.222 (0.034) | 3.3 3.1 | 0.171 (0.040) | 2.9 2.3 |
| United Kingdom 1953-88 | 0.234 (0.070) | 5.6 4.9 | 0.120 (0.055) | 4.6 3.6 |
| France 1952-88 | 0.083 (0.042) | 3.9 4.4 | 0.0075 (0.059) | 3.4 2.9 |
| Italy 1953-88 (M1) 1955-88 (M2) | 0.203 (0.063) | 5.4 4.1 | 0.139 (0.051) | 4.4 3.5 |
| Canada 1950-88 | 0.552 (0.115) | 5.8 5.4 | 0.134 (0.066) | 4.5 3.8 |

Standard errors are printed behind each interest rate elasticity. All "official" and robust estimates of the standard error of the forecasts are in percent.

rate elasticity of M2 is smaller than that of M1, except in the U.S. where the elasticities are estimated to be about equal. In five of the seven countries the interest rate elasticities for M1 are quite close together (United States, Japan, Germany, United Kingdom, Italy). The elasticities are higher but still of the same order of magnitude as found in earlier work by Den Butter and Fase (1980).

Table 1 also shows the size of the forecast errors. These are conditional on the realized value of the long-term domestic bond yield and the estimated interest rate elasticity and on the optimal estimates of the relative importance of the three different types of shocks that affect velocity as well as on the covariance between the permanent shocks to the level and the permanent shocks to the growth rate. As far as the intercept and the trend in velocity are concerned, the forecasts are purely ex ante and computed recursively without any smoothing.

The reasons for computing the forecasts conditional on the interest rate for the current year are twofold. First, the outcomes are directly comparable to results from studies of the demand for money, the principal differences being that the Kalman filter does not require knowledge about future shocks and allows for a stochastic trend in velocity. Second, because interest rates are observed without lag and without measurement error, policymakers can always adjust any targets for a monetary aggregate if interest rates during the planning period deviate from their predicted values when the targets were set. Hence, one could argue that forecasts conditional on interest rate realizations produce more useful evidence about the forecastability of velocity than forecasts that are conditional only on past values of velocity, income and interest rates.

Table 1 gives two estimates of the accuracy of the forward Kalman filter. The first number for each country and each monetary aggregate indicates the root mean square error of the forecasts for the period as indicated. The second number is a robust estimate of that same root mean square error, computed using the median absolute deviation divided by the correction factor 0.6745. For normally distributed values this robust estimate has the same expectation as the standard error. Outliers in the series cause the robust estimate to be smaller than the "official" standard error.

The table confirms that outliers are important in several countries. The following is a list of all outliers, defined as forecast errors (in percent) in excess of three times the robust estimate of the standard error of the forecasts for the country and aggregate concerned:

|  | M1-velocity | M2-velocity |
|---|---|---|
| <u>Japan</u> | 1971: -16.2 | 1971: -11.2 |
| <u>Germany</u> | -- | 1960: 9.3 |
| <u>United Kingdom</u> | 1963: -16.3 | 1972: -14.5 |
| <u>France</u> | -- | 1987: -10.9 |
| <u>Italy</u> | 1960: 12.7<br>1970: -13.5<br>1974: 14.2 | 1960: 13.5 |
| <u>Canada</u> | -- | 1954: -11.1<br>1983: 14.9 |

Note that two of the twelve outliers relate to years in the period 1980-1988, which does not support the hypothesis that outliers became more frequent in the recent period.

The figures show that in almost all cases the trend does vary over time. The exceptions are M2 in the United States, M1 in Germany and M2 in Japan where the changes in the trend are economically insignificant. For these three cases the forecasts do not become significantly less succesfull if a constant trend is used. In the other models, succesfull forecasting does require a trend that can change over time, i.e. a value for the variance of $w_2$ that is different from zero.

Figures 1-7 show the patterns over time of the stochastic trends in velocity of M1 and M2 as estimated by the smoothed annual filters.

### IV.   Has Velocity Become More Unpredictable?

This section discusses a number of  additional hypotheses regarding the unpredictability of velocity.  Table 2 presents results of a formal test whether the forecasts of velocity have become more imprecise in the 1980s. For each country the two numbers in each cell in the table refer to the variance of the forecast errors over the period through 1979 and the variance of the forecast errors over the period 1980-89.  Forecasts errors

Figure 1

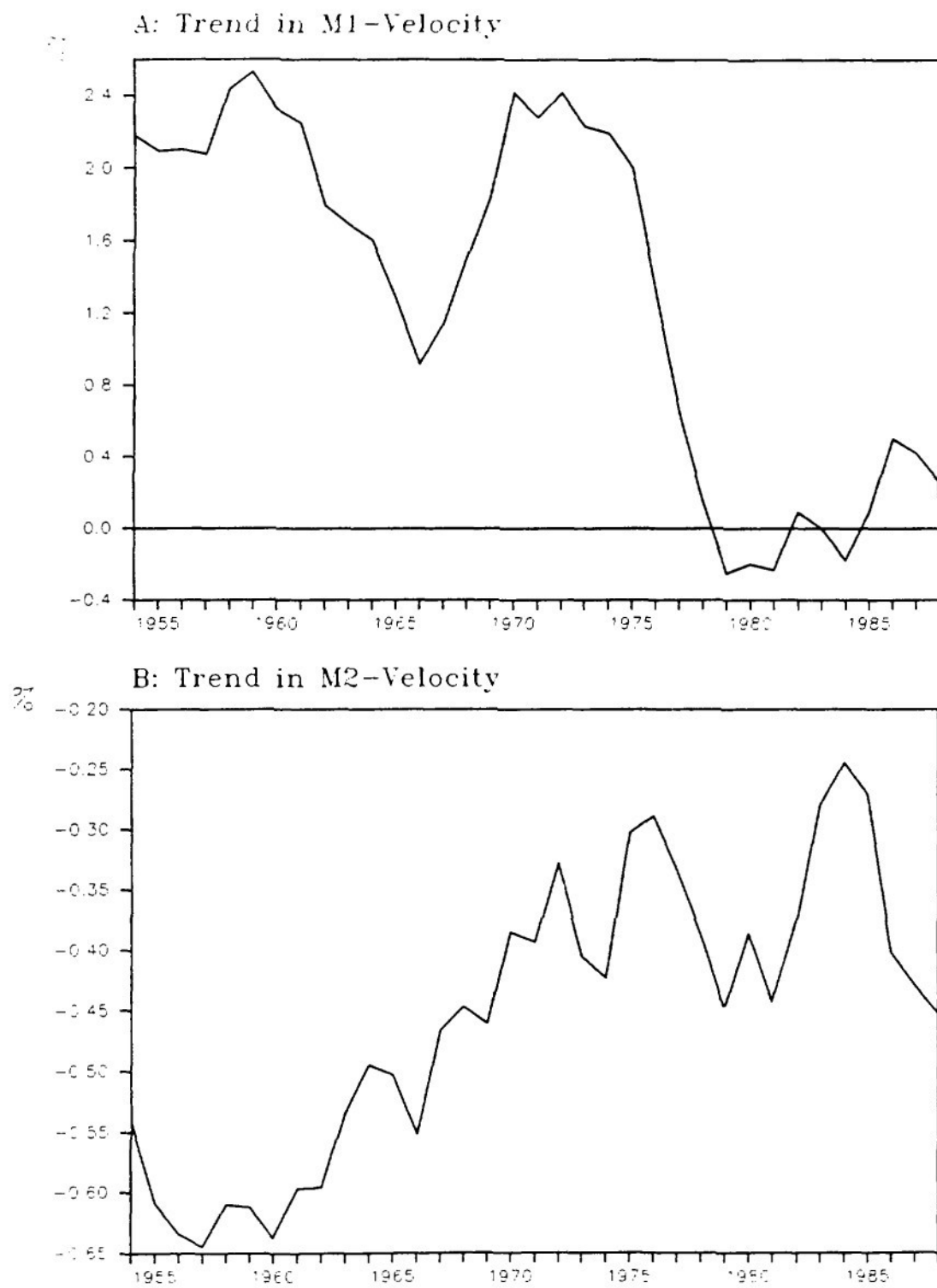Smoothed Trend in Velocity in the United States

## A: Trend in M1-Velocity



## B: Trend in M2-Velocity
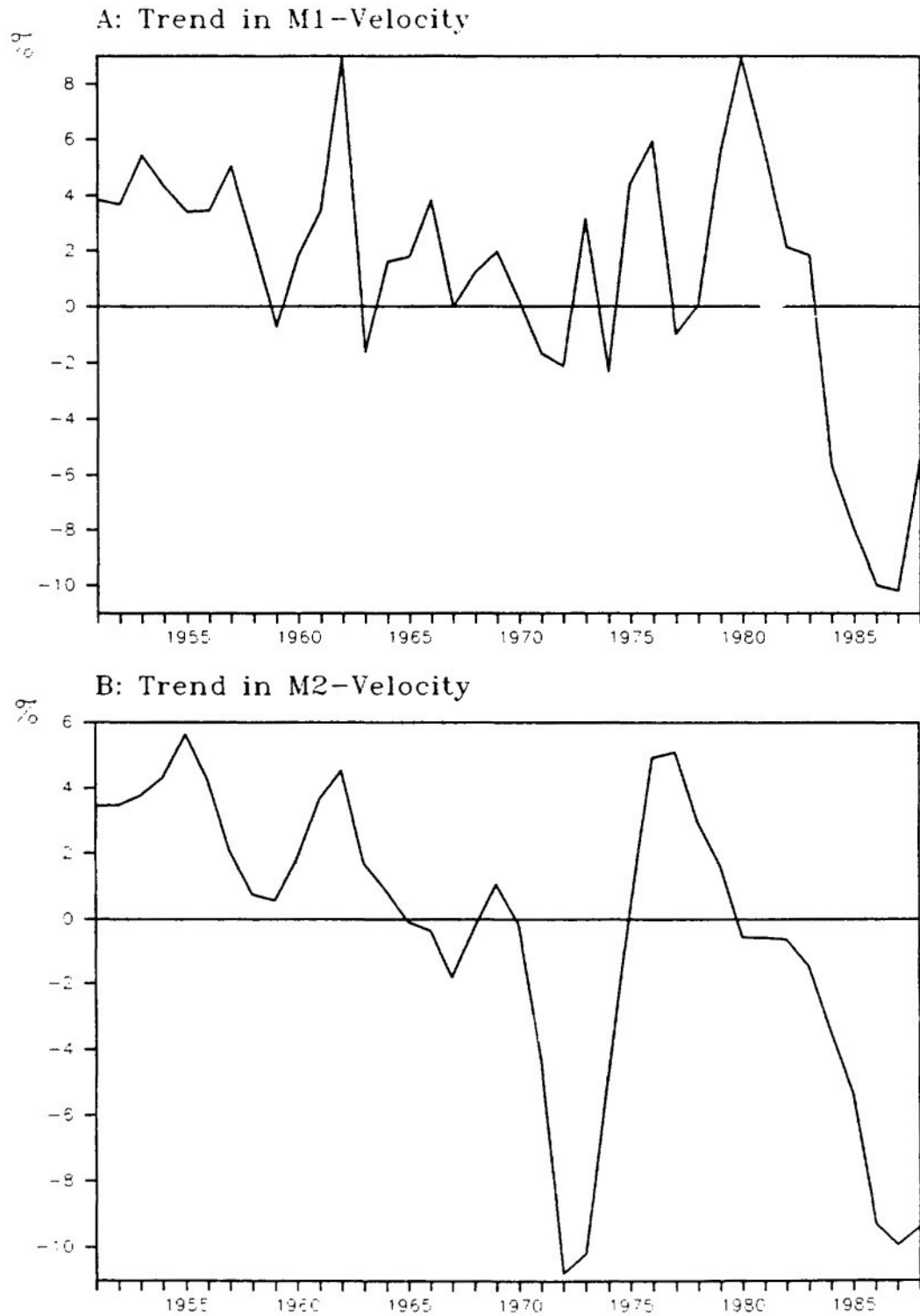
Figure 2

Smoothed Trend in Velocity in the United Kingdom

## A: Trend in M1-Velocity



## B: Trend in M2-Velocity

Figure 5

Smoothed Trend in Velocity in France

## A: Trend in M1-Velocity



## B: Trend in M2-Velocity
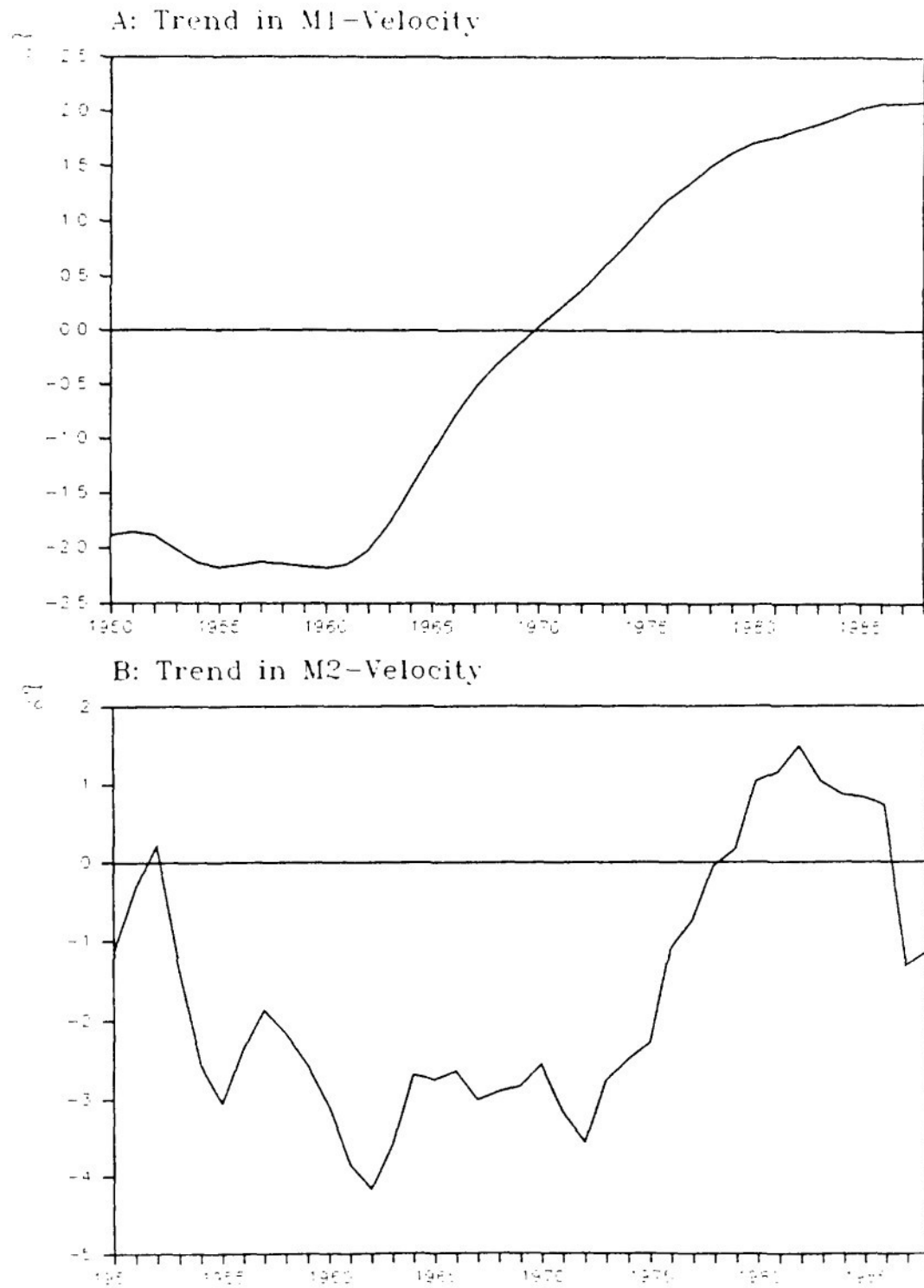
Figure 4

Smoothed Trend in Velocity in Germany

## A: Trend in M1-Velocity



## B: Trend in M2-Velocity

Figure 5

Smoothed Trend in Velocity in Italy
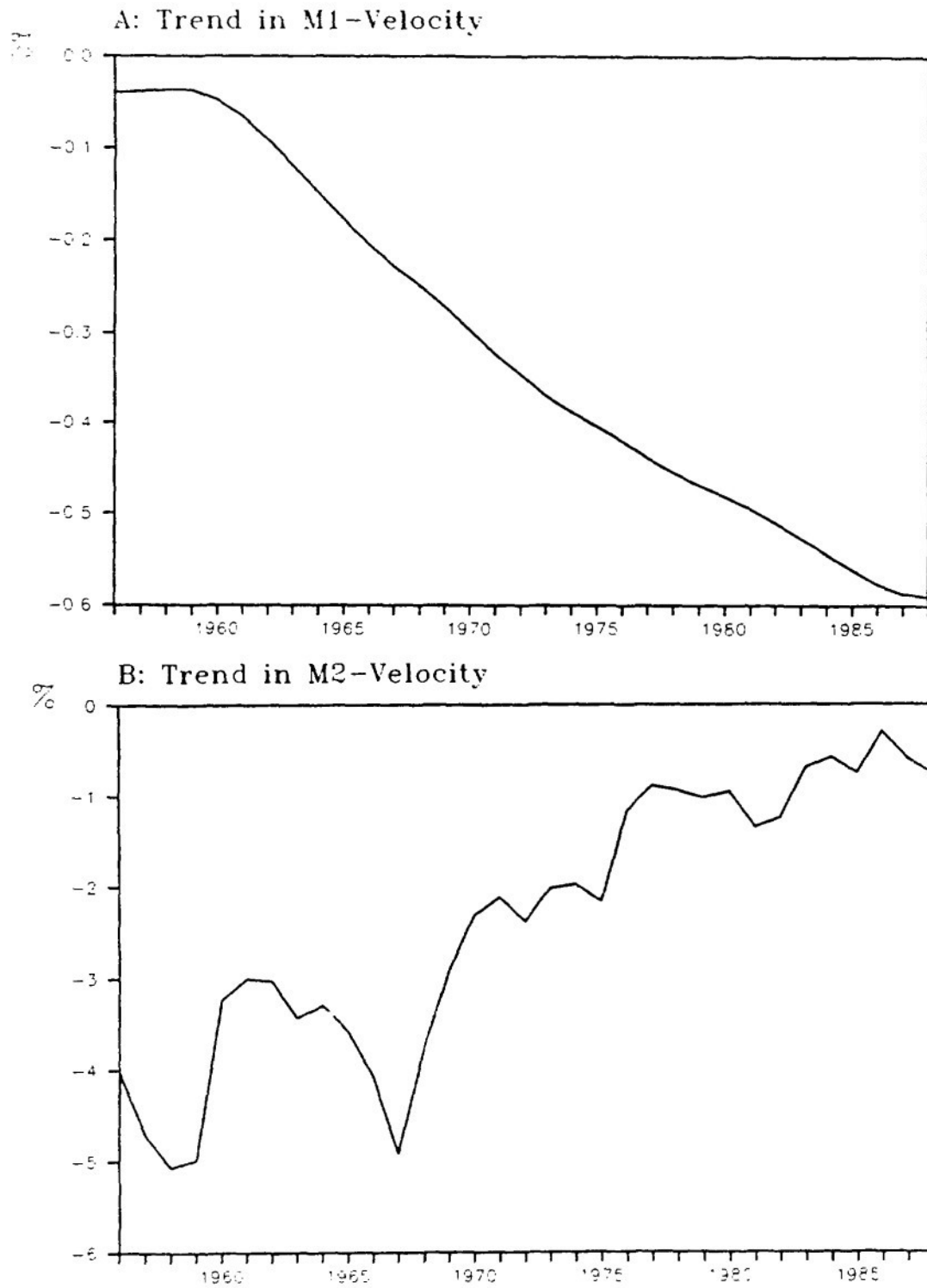
A: Trend in M1-Velocity



B: Trend in M2-Velocity
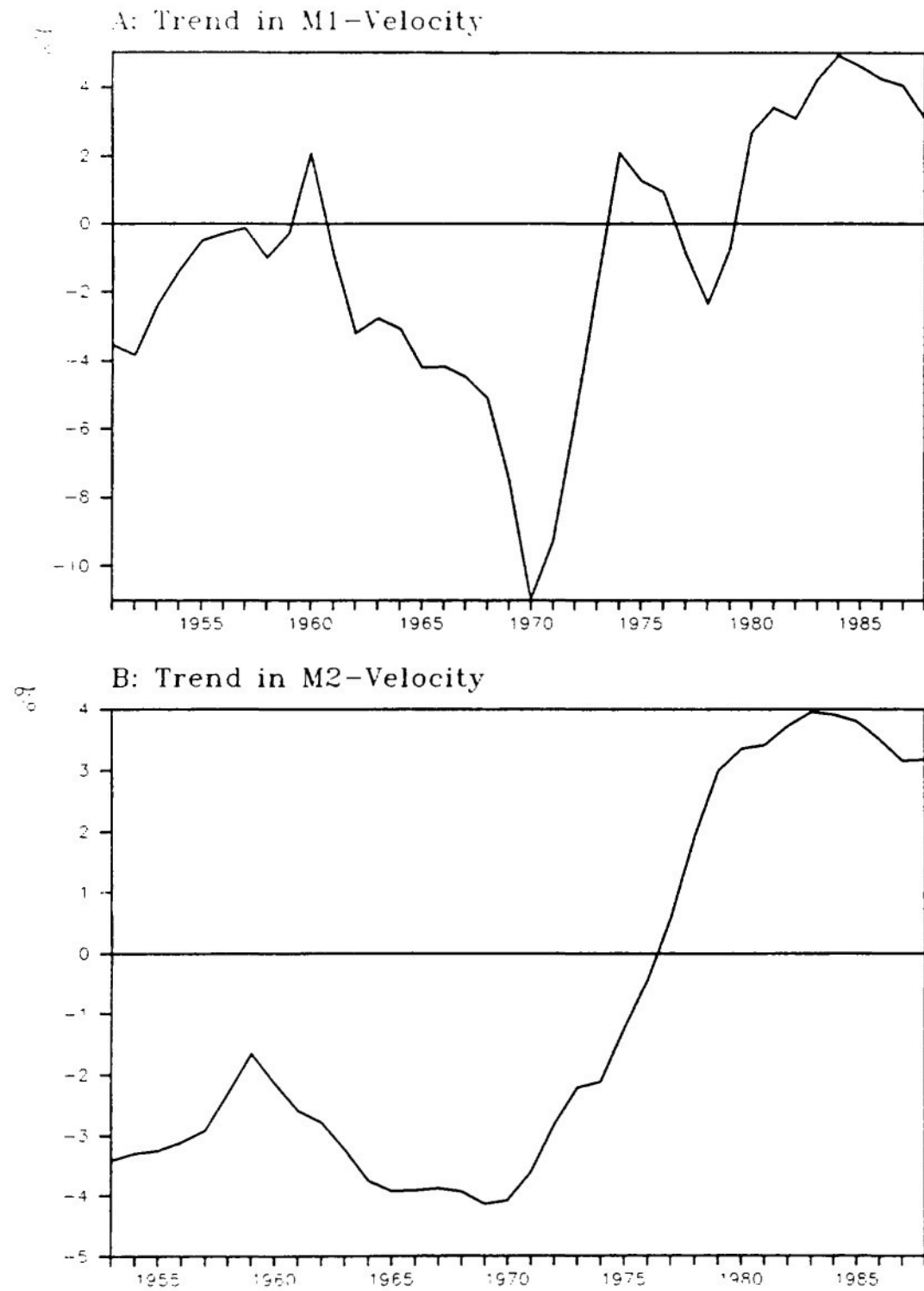
Figure 6

Smoothed Trend in Velocity in Canada

## A: Trend in M1-Velocity



## B: Trend in M2-Velocity
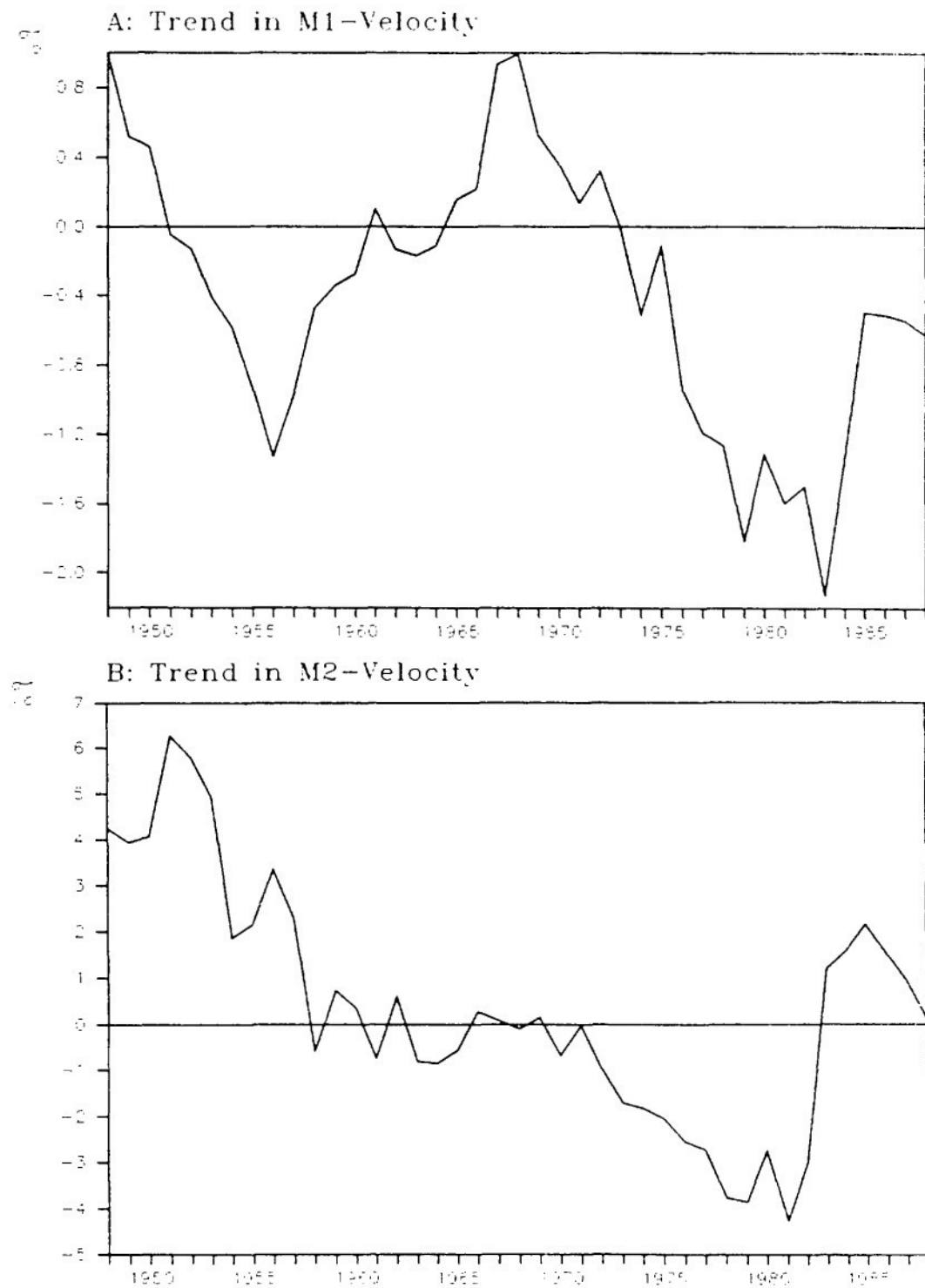
Figure 7

Smoothed Trend in Velocity in Japan

A: Trend in M1−Velocity



B: Trend in M2−Velocity
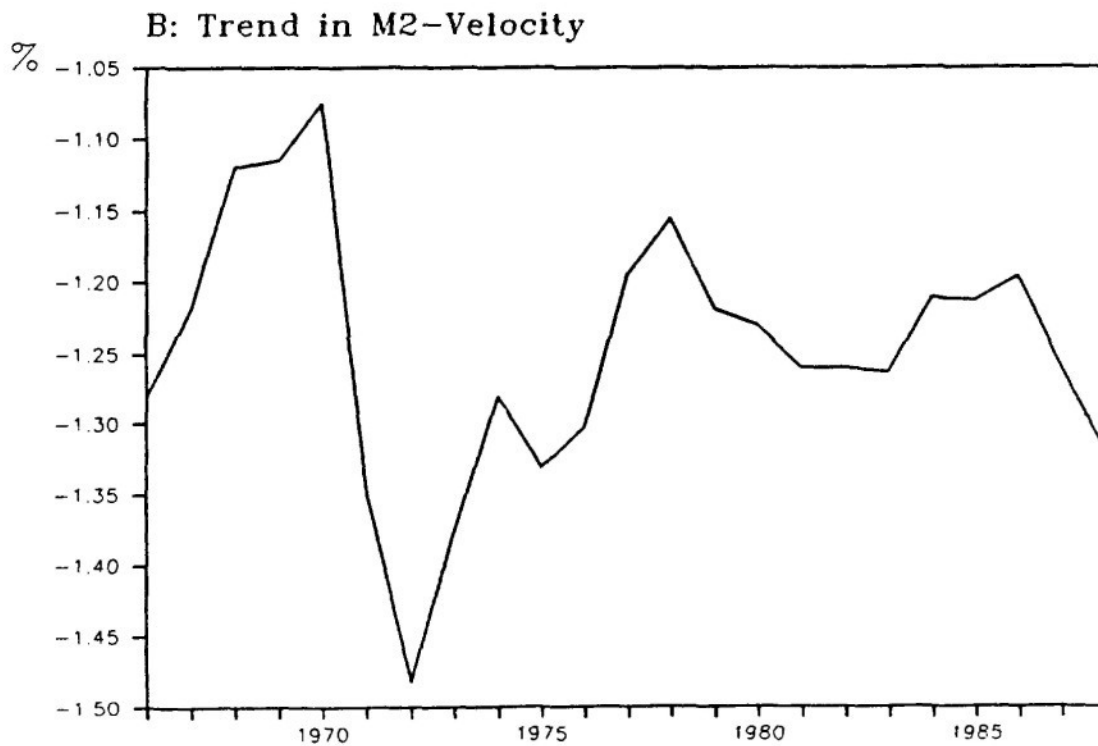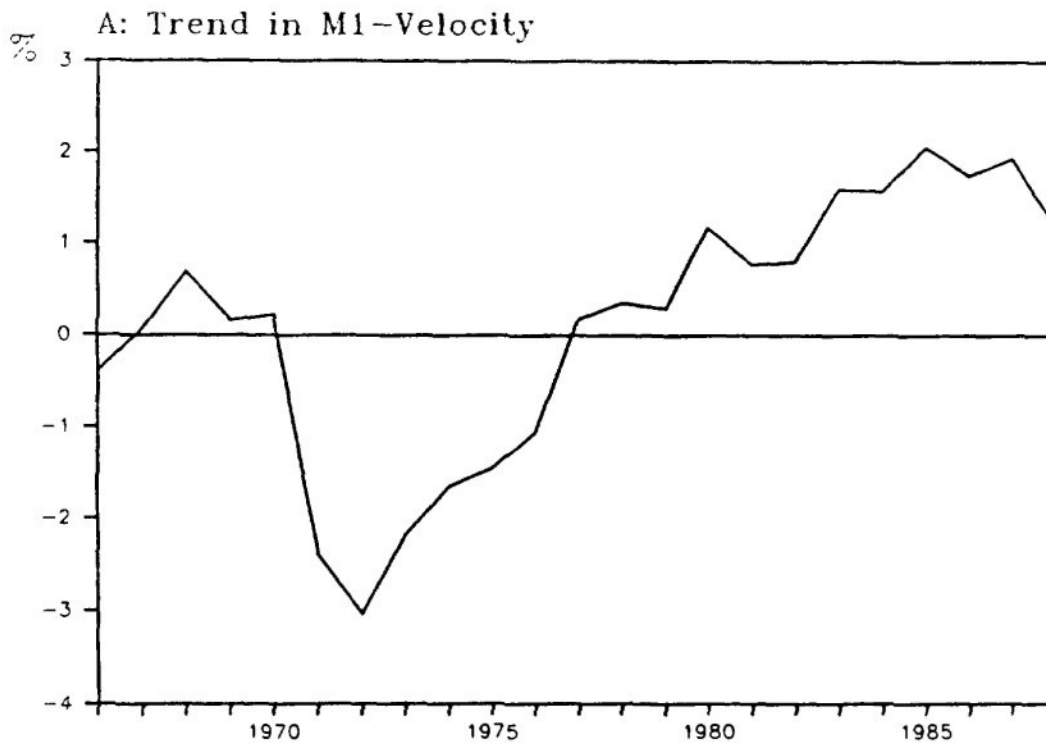
Table 2.  Changes in Forecast Errors of Velocity

|                | M1          | M2          |
|----------------|-------------|-------------|
| United States  | 4.8 / 10.2  | 4.1 /  8.3  |
| Japan          | 40.8 / 13.1 | 23.4 /  2.4 |
| Germany        | 12.7 /  8.9 | 11.5 /  3.1 |
| United Kingdom | 30.9 / 28.5 | 24.9 / 16.3 |
| France         | 18.4 /  7.0 | 10.3 / 19.1 |
| Italy          | 33.7 / 19.2 | 20.1 / 20.8 |
| Canada         | 25.5 / 69.8 | 17.0 / 38.7 |

All numbers must be multiplied by $10^{-4}$.  The first number in each pair refers to the variance of the forecast errors before 1980; the second number to the variance over 1980-88.  Forecast errors are based on an on-line Kalman Filter that uses the current value of the long-term government bond rate.

are taken from the final forward filter as discussed in Section II and use
the current realization of the interest rate. The sum of squared forecast
errors has been divided by n-1, with n the number of errors in the sample.

The results in Table 2 reject the notion that the income velocity of
money has become more unpredictable worldwide in the 1980s. The errors
become larger in the United States and in Canada, as well as for M2 in
France, in each case by a factor of 2 or slightly above. On a formal F-test
this is insufficient in all five instances to reject the null-hypothesis
that the variance of velocity has remained unchanged. Velocity of M2 in
Italy is as predictable before 1980 as after that year. In all eight other
cases, the forecast errors decline, sometimes by a very large margin.

When the variances for both aggregates and both periods are arranged in
order of magnitude across the countries, one sees that for M1 velocity the
median value of the variance falls from 25.5 to 13.1 and for M2 from 17.0 to
16.3. It is also interesting to note that before 1980 M2 velocity was less
variable than M1 velocity in all seven countries, and during the 1980s in
five of the seven countries. Particularly small are the forecast errors for
M2 velocity in Japan and Germany.

Table 3 shows how the Kalman filter forecasts compare with an
alternative method of generating forecasts of velocity. Regressions have
been performed for both monetary aggregates according to the following
specification:

$$(4) \qquad V_t = c + \alpha tr_t + \beta y_t + \theta i_t + u_t$$

$$u_t - \phi u_{t-1} = a_t$$

In equation (4), velocity is regressed on a linear trend, real income, and
the long-term interest rate. A first-order autoregressive parameter is
estimated for the residuals in all 14 cases. These regressions are updated
each year and the regression coefficients are used for forecasts conditional
on the realized values of real income and the interest rate, and again
incorporating the serial correlation correction. The right hand side of the
table indicates the forecast errors for the years 1980-88. The numbers in
the table show the mean errors (the bias) and the standard deviations of the
conditional forecasts for 1980-1988. For purposes of comparison, the
forecasts with the Kalman filter are also purely ex ante. The parameters of
the Kalman filter were re-estimated for the period ending 1979 and the model
was extrapolated for 1980-1988. Hence, all numbers relate to ex ante
forecast errors, conditional on the realized values for velocity and the
interest rate, with the log of income as an additional variable in the

Table 3.  Forecast Errors for 1980-88

(In percent)

| | | Kalman filter | | Regressions | |
|---|---|---|---|---|---|
| | | bias | standard error | bias | standard error |
| United States | M1 | -1.82 | 2.10 | 2.48 | 2.15 |
| | M2 | 0.13 | 2.69 | 0.55 | 4.71 |
| United Kingdom | M1 | 0.11 | 2.56 | 5.47 | 6.97 |
| | M2 | 0.11 | 1.89 | 6.06 | 4.52 |
| France | M1 | 0.25 | 2.54 | -4.53 | 4.46 |
| | M2 | -0.93 | 3.99 | -4.86 | 6.68 |
| Italy | M1 | 1.35 | 2.90 | -4.91 | 3.50 |
| | M2 | 0.45 | 4.28 | -3.30 | 5.21 |
| Germany, Fed. Rep. of | M1 | -0.24 | 2.79 | -0.98 | 3.74 |
| | M2 | 0.05 | 1.75 | -0.37 | 1.84 |
| Canada | M1 | -3.13 | 6.92 | 3.40 | 9.99 |
| | M2 | 1.80 | 5.68 | -3.64 | 5.70 |

rolling regressions. 1/ The Kalman filter gives less biased forecasts with far lower forecast errors.

Finally, Table 4 investigates whether the differences in predictability of velocity across countries are related to the unpredictability of the money supplies. Identical Box-Jenkins time-series models have been applied to the money supply data, assuming a first-order moving average model applied to the second differences of money stock data. The table shows the estimated standard errors of the fourteen Box-Jenkins models, together with the robust estimates of the standard errors of the forecasts in velocity. Spearman's rank correlation coefficients have been computed between these errors and the forecast errors for velocity. For both M1 and M2 the rank correlation coefficient equals 0.71 which is at the 0.05 significance level.

Table 4 also investigates the rankings of the forecast errors in money and in velocity on a cross-sectional basis. One can also rank the forecast errors for money and velocity in each country in order to see whether years in which realized money growth deviated much from predicted money growth tended to be years in which velocity also deviated substantially from its conditional forecast.

Significant rank correlations at the 0.05 level, using Spearman's method, are obtained in the following cases: United Kingdom M1 (coefficient=0.55) and M2 (0.64), France M2 (0.32), Italy M1 (0.49) and M2 (0.36), and Canada M1 (0.49) and M2 (0.62). The t-value for $\rho$ for M1 in Japan equals 1.76 (significant at the 0.10 level). Hence, in the three countries in which M1 or M2 velocity was most variable, years with large forecast errors in money tended to be years with large forecast errors in velocity.

## V.  Conclusions

The Kalman filter results indicate a substantial value of the interest rate elasticity of the demand for M1. Niskanen (1988) seems to have been the first to indicate that some earlier estimates of the demand for real balances in the United States might have gone astray by assuming that the secular increase in velocity during the 1970s should be represented by a linear trend. He pointed to the alternative hypothesis that the demand for money fell during that period because of higher trending interest rates, an insight that was supported by Poole (1988). Poole's paper describes why a substantial interest rate elasticity makes the conduct of a disinflationary monetary policy more difficult: the rate of growth of the money supply has to decline in order to lower inflationary expectations, but as the lower inflationary expectations lead to lower long-term interest rates, the demand for real balances goes up.

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1/ The Kalman filter model for Japan was not re-estimated for a shorter period, because the sample starts only in 1966.

Table 4.   Forecast Errors of Money and Velocity

(In percent)

|  |  | M1 | (rank) | M2 | (rank) |
|---|---|---|---|---|---|
| United States | velocity | 2.59 | (1) | 2.28 | (1) |
|  | money | 2.75 | (1) | 3.06 | (4) |
| United Kingdom | velocity | 4.87 | (6) | 3.55 | (6) |
|  | money | 5.36 | (5) | 5.20 | (7) |
| France | velocity | 4.37 | (5) | 2.93 | (4) |
|  | money | 3.48 | (2) | 2.85 | (2) |
| Germany, Fed. Rep. of | velocity | 3.12 | (2) | 2.28 | (2) |
|  | money | 4.02 | (3) | 2.17 | (1) |
| Italy | velocity | 4.09 | (3) | 3.47 | (5) |
|  | money | 4.42 | (4) | 3.13 | (5) |
| Canada | velocity | 5.42 | (7) | 3.80 | (7) |
|  | money | 6.55 | (7) | 4.36 | (6) |
| Japan | velocity | 4.25 | (4) | 2.71 | (3) |
|  | money | 5.37 | (6) | 2.96 | (3) |

The results lend no support to the hypothesis that the income velocity of money has become significantly more unpredictable in the 1980s. Forecast errors did increase in the United States but became smaller in most other countries. The frequency of outliers, defined as particularly large forecast errors, also did not increase during the years 1980-88. There is a significant correlation between the size of the forecast errors in velocity and the size of the forecast errors in money: predictable monetary policies are associated with predictable behavior of velocity.

Finally, regarding methodological issues, the Kalman filter allows one to specify the model in terms of levels, even though the dependent variable, the explanatory variables and the error terms are nonstationary. The levels specification has important advantages: smaller measurement errors in the dependent (and independent) variables, and superior estimates of the coefficients if the independent variables affect velocity with a variable lag.

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